An Anatomy of the Repo Market Crash

Wei Qiao*

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Abstract

The repo market crash was a catalyst for the great recession in 2008-2009. I evaluate the quantitative importance of the following three factors in that crash: a drop in the price of residential mortgage-backed securities (RMBS), the liquidity drying up caused by asymmetric information in the RMBS market, and the run by repo lenders induced by changes in the fundamentals. On the theoretical side, the main contribution is to construct a tractable and parsimonious model to integrate the RMBS market with asymmetric information and the repo market with strategic complementary lenders. The two markets are connected by buyers in the RMBS market who use RMBS as collateral for borrowing in the repo market. I characterize the stochastic equilibrium of the economy where the quality of RMBS follows a Markov process. With calibration and simulation, the model yields the following quantitative results. First, the liquidity drying up caused by asymmetric information plays a crucial role in every aspect of the repo market crash. It explains 30% of the increase in haircut, 13% of the drop in total repo outstanding, and a large part of the increase in repo spread. Second, throughout the crisis, the fundamental-based run significantly affects the repo rate but only has a small effect on the repo haircut. Third, in addition to the three factors, the general equilibrium effect generated from the interactions between the RMBS market and the repo market explains 33% of the drop in total repo outstanding. I discuss the policy implications of these findings.

*Job market candidate at the economics department of the Pennsylvania State University. Comments and suggestions are welcome. Email: wxq7@psu.edu

1 Introduction

Marked by the largest bankruptcy filing in U.S. history, Lehman Brothers failed to roll over billions of dollars on the repo market[1] because of its significant exposures to the residential mortgage-backed securities (RMBS).[2] The Lehman Brothers' failure offered a vivid example, in miniatures, of how do interactions between the RMBS market and the repo market drive the insolvency of the wholesale banking sector, which led to the Great Recession.[3] This paper constructs an equilibrium model to capture those interactions. The goal is to use the model to quantitatively evaluate contributions of three factors (described below) that are commonly believed to be important for the repo market crash. I also explore the policy implications of findings from the quantification exercise.

Three factors are proposed to explain the repo market crash: the price factor, the liquidity factor, and the (fundamental-based) run factor. Each of them is rooted in the institutional details of the repo market. First, notice that a significant portion of collateral assets used in the repo contract are RMBS,[4] of which the security design makes their return very sensitive to the housing price fluctuation. Observing the unprecedented housing price decline in 2007, repo lenders may be worried about the qualities/prices of their collateral assets and withdrawal cash from the repo market. I will call it the price factor in the following. Second, the repo market crash may also be triggered by repo lenders' concern about the liquidity of RMBS. It is because RMBS are very complicated structural financial products. The complicity generates asymmetric information between RMBS sellers and buyers when its fundamentals keep changing (e.g., the default probability of the subprime mortgages), which potentially impedes liquidity. In the rest of the paper, this factor will be referred to as the liquidity factor. Third, the repo market crash may be a consequence of the maturity mismatch between repo borrowers' long-term investment in RMBS and short-term repo liability, making them particularly vulnerable to runs by repo lenders. I will exclusively focus on the "fundamental-based run" instead of the "panic-based run" since "panics" is impossible to measure and

[1]Repo market is a short-term collateralized borrowing market mainly participated by dealer banks and cash-rich institutions like money market funds. The appendix A contains a brief review of institutional backgrounds and financial terminologies used in the paper.

[2]See the detailed case study by Wiggins et al. (2014).

[3]For example, see Gorton and Metrick (2012) and Brunnermeier (2009).

[4]In this paper, I mainly focus on the bilateral repo market that is associated with the private-labeled RMBS as collaterals. As suggested in Gorton and Metrick (2012), this is the part of the repo market that is considered as the culprit of the Great Financial Crisis.

quantify. I delay the detailed discussion of this choice to the literature section. I denote the (fundamental-based) run as the run factor. This paper evaluates the quantitative importance of the price, the liquidity, and the run factor in the repo market crash.

The RMBS market and the repo market are tightly connected. Buyers in the RMBS market use previously purchased RMBS as collateral for borrowing in the repo market. Therefore, decoupling the three factors is a challenging task. They are inevitably interwoven with each other. First of all, prices and liquidities of RMBS are typically simultaneously determined on the market. Therefore, the price factor naturally correlates with the liquidity factor. Secondly, regulated by accounting rules, collateral assets are marked to market. When repo lenders decide whether to run the repo borrower or not, they will consider the liquidation values of their collateral portfolio. Thus all three factors are dependent on each other. Thirdly, all three factors may correlate across time because of the endogenous evolution of repo borrowers' balance sheets and budgets. Given the limited data availability of repo market transactions, it is impossible to account for these endogeneities and decompose the three factors from each other without a structural model.

Unfortunately, to the best of my knowledge, a suitable model for answering my research question does not exist in the literature. The difficulties are the following. Capturing the price factor implies the model must be stochastic. Asymmetric information friction, the key to the liquidity factor, requires the model to include heterogeneity in the RMBS market. And the run factor calls for tracking the dynamics of a repo borrower's balance sheet. It is well known that a dynamic stochastic model with heterogeneity is hardly tractable. So on the theoretical side, the contribution of this paper is that I construct a parsimonious model which nests the price factor, the liquidity factor, and the run factor under a unified framework and tracks their interactions. I propose a simple concept of equilibrium and characterize its properties.

In my model, there are three agents: sellers, buyers, and lenders. They interact in two markets: sellers and buyers trade assets in the asset market, and buyers borrow cash from lenders in the repo market. There are multiple types of assets ordered by their qualities. I assume that the quality of the lowest type asset follows a Markov chain, which comes from housing price fluctuations. The stochastic quality changes capture the price factor. Furthermore, I assume asset types are only observable to sellers, addressing the liquidity factor caused by the asymmetric information friction. The asset market is modeled á la a stochastic version of Guerrieri and Shimer (2014). Given buyers' budget, prices and liquidities of all types of assets are endogenously determined in the equilibrium. In the repo market, buyers propose take-it or leave-it repo contracts to lenders, and all lenders simultaneously choose between the repo contract and exogenous heterogeneous outside options. Two conditions constrain repo contracts. (1) Buyers cannot promise more asset collateral than their available asset holdings, and (2) the liquidation value of collateral portfolios, evaluated by the equilibrium price and liquidity on the asset market, must exceed the face value of the repo contract. Notably, there is a strategic complementarity term in lenders' utility functions depending on how many other lenders prefer repo contracts. This term is essential for the run factor. The simultaneous-move game among lenders is modeled akin to Morris and Shin (2001). The asset market connects with the repo market through the budget balance condition of buyers. Buyers' decisions in two markets endogenously determine the evolution of their balance sheets, of which all agents have rational expectations. The equilibrium framework captures the rich dynamics and interactions among the three factors.

Beyond theoretically tracking the interactions of the three factors using the structural model, quantitatively evaluating their contribution is also important and interesting. Each of the three factors has straightforward but distinct policy implications. If the price factor is the most important, bailing out buyers can be costly and may exacerbate their risk-taking behaviors in the long run. Also, it implies the security design of RMBS and other structured financial products need to be regulated after the crisis. If the liquidity factor is the most important, however, bailing out buyers that are in trouble is costless and beneficial for financial stability. Large asset purchase programs that boost the liquidity of the asset market are effective and may generate profits. In the long run, the government also needs to take measures on the information disclosure and regulate the credit rating industry to prevent the crisis. If the run factor is the most important, FED acts as a last resort lender, and the arrangement similar to the deposit insurance is the most effective tool. Macroprudential policies such as increasing the reserve ratio of repo borrowers are recommended. Notice that all these policies have different trade-offs and sometimes contradict each other. Answering my research question is crucial for policy evaluation during the crisis and the regulation reform after it. Moreover, the importance of each factor may change over time, shedding light on the timing of different interventions.

Given the importance and the interest in the quantitative result, another contribution of this paper is that I solve the model numerically and calibrate it to data using the standard simulated method of moments. I discipline the three factors with three sources of information. For the price factor, I estimate the stochastic movements of asset qualities to match the ABX index. For the liquidity factor, I use the loss distribution of private-labeled RMBS reported by Ospina and Uhlig (2018) to calibrate the distribution of asset supplies in different qualities. Strategic complementarities among lenders are not observable. I back it out indirectly. The equilibrium framework endogenously decomposes the repo spread into four terms: a term related to the over-collateralization, a term that reflects the buyer's credit risk, a term that illustrates the strategic complementarity among lenders, and a term that accounts for the heterogeneous outside options. To identify the parameter of strategic complementarity, I simultaneously target the repo spread, the haircut rate (a measure of the over-collateralization), and the LIB-OIS spread (a proxy of buyer's credit risk) published in Gorton and Metrick (2012).

With the calibrated model, I conduct counterfactual experiments to decompose the price, the liquidity, and the run factors. Three results deserve special attention. Firstly, I found the liquidity drying up caused by asymmetric information plays a crucial role in every aspect of the repo market crash. It explains 30% of the increase in haircut, 13% of the drop in total repo outstanding, and a large part of the increase in repo spread. Secondly, throughout the crisis, the fundamental-based run has a significant and persistent effect on the repo rate but only a tiny impact on the repo haircut. Thirdly, the general equilibrium effect generated from the interactions between the RMBS and the repo markets explains 33% of the drop in total repo outstanding. This result, in hindsight, confirms that it is vital to study the RMBS market and the repo market together rather than separately.

In summary, my results provide structural evidence for the narrative in Gorton (2009). The unfolding of the repo market crash is likely to be the following. First, it started from quality drops of a subset of subprime RMBS. These drops were amplified by the asymmetric information friction on the RMBS market and had detrimental effects on the liquidities of those securities. Worrying about the volatility of liquidation values of their collaterals, lenders required higher and higher haircuts for repo borrowing. With less access to repo fundings, the demand for RMBS declined significantly and further depressed both RMBS prices and liquidities. Finally, the repo market fell into massive turmoil because of panics among lenders, probably triggered by the failure of Lehman Brothers.

These results imply bailing out the troubled banks during the great recession is not as costly as previously believed. My results support various liquidity programs initiated by the FED during the crisis and predict the effectiveness of those interventions in mitigating the increase of haircuts. However, my second quantitative result suggests that conventional monetary policies are ineffective in easing the haircut surge. Federal fund rate cuts increase the repo spread, which has little effect on the haircuts. Notice that the ineffectiveness is independent of zero lower bounds. I also investigate the impact of macro-prudential policies such as compulsory cash reserve in section 8.

2 Literature

Quantitative evaluations of the three factors require a model that integrates the two markets, where asset buyers act as the main connection. In this section, I compare each component of my model with their counterparts in the literature and discuss the value-added.

The price factor is closely related to the ”originate and distribute” explanation for the Great Recession. This narrative emphasizes the incentive structure implied by the wholesale banking system. The main idea is summarized by the principal-agent theory. The agent (the originator of the loans) did not have the incentives to act fully in the interest of the principal (the ultimate holder of the loan). Originators had every incentive to maintain origination volume because that would allow them to earn substantial fees, but they had weak incentives to maintain loan quality. The declining underwriting standard of the subprime mortgage loans before the crisis is documented by several empirical studies such as Keys et al. (2008), Keys et al. (2009) and Keys et al. (2012). However, declining underwriting standards does not necessarily imply that the quality of the mortgage-backed security is low, considering that these securities are specifically designed to hedge the potential increasing default risk. Indeed, from a retrospective view, Ospina and Uhlig (2018) has documented that the AAA-rated subprime mortgage-backed securities have better performance than the AAA-rated prime mortgage-backed securities. Furthermore, even if some of the mortgage-backed securities have experienced a significant quality drop, how much it contributes to the repo market crash is unclear. My paper takes this literature as the micro-foundation of the price factor and contributes to it by evaluating its quantitative significance.

There are also many other authors focusing on asymmetric information friction in the structural financial products market. Some of the related works are Guerrieri et al. (2010), Guerrieri and Shimer (2014), Guerrieri and Shimer (2018), Chang (2018), and Williams et al. (2016). All these papers embed asymmetric information friction with single or multi-dimensional private information into a standard directed search model (in discrete or continuous-time). A common feature is that buyers and sellers endogenously use prices and liquidities as signaling/screening devices. The more severe the asymmetric information friction is, the lower the liquidities and prices are. My model shares a comparable framework and has similar theoretical implications. However, there are important differences. The first difference is that buyers in all papers above either have a fixed budget or deep pocket, which eliminates the role of demand fluctuation in the asset market equilibrium dynamics. Hence they are unable to capture the feedback between the asset market and the repo market. My model allows buyers' total budget of purchases endogenously determined within the framework and has nontrivial interaction with asset prices and liquidities. The second difference is that to capture the price factor, my model follows a stochastic setup. I have generalized and confirmed that the solution techniques in the literature, with some modifications, also work in the stochastic case.

The lenders' run is quite similar to the run on the traditional banking sector. Models in this category can be divided into two subgroups. The first is that the crisis is attributed to the coordination failures among lenders such as Diamond and Dybvig (1983). This kind of run relies on the existence of the multi-equilibrium. The probability of a run is determined outside the model. The other type of run is more often referred to as "fundamental-based" run. Morris and Shin (2001) and Goldstein and Pauzner (2005) are two representatives. They introduce small noises to relevant fundamentals perceived by agents. This small incomplete information friction, strikingly, leads to the selection of a unique equilibrium, despite agents are strategically complementary to each other. The probability of a run is determined endogenously. For my goal of the research, I follow Morris and Shin (2001) for the modeling of the repo market. An alternative model is He and Xiong (2012), which analyzes a dynamic coordination problem and also obtains uniqueness. The main difference between my model and theirs is that they (and the most of papers with similar models) take the relevant fundamentals as an exogenous stochastic process. My paper, however, connects the economics fundamentals with asset markets. Lenders' payoffs are jointly determined by not only other lenders' actions but also prices and liquidities determined on the asset market.

This paper also completes the literature on the explanation of the Great Recession. From the theoretical perspective, Bernanke and Gertler (1989), Bernanke and Gertler (1990) and Kiyotaki and Moore (1997) pioneer the research on the dynamics of collateral values, the agents' ability to borrow and the business cycle. For its application, to mention a few, Jermann and Quadrini (2012), Shi (2015), and Del Negro et al. (2017) test and evaluate how do financial shocks, together with different mechanisms, explain (or fail to explain) the Great Recession. While they connect the financial shocks with the business cycle fluctuation, my model supplements the picture by providing a theory illustrating how do housing market fluctuations develop to a financial market turmoil, explaining the source of these financial shocks. Brunnermeier and Pedersen (2009), Martin et al. (2014) and Wang (2019) consider the related hypothesis of the repo market crash. Their models either feature finite periods or are hard to calibrate. My model is the only one that generates quantitative implications.

3 Model

3.1 The Model Environment

Time, denoted by t, is discrete and infinite. There are three types of agents in the model: asset sellers (henceforth abbreviated as sellers), asset buyers who purchase assets, and at the same time borrow cash via repo contract (henceforth abbreviated as buyers/borrowers), and lenders who lend cash to buyers. Each type contains a continuum of risk-neutral individuals.[5] The aggregate measure for sellers and lenders is one. There is a unit measure of buyer families. Within each buyer family, there exists a continuum of individual buyers with measure one. Agents interact with each other in two markets: an asset market where sellers and buyers trade assets and a repo market where buyers borrow cash from lenders by posting their asset holdings as collaterals. Let ρ^l denote the discount factor for sellers, and ρ^h with $0 < \rho^l < \rho^h$ be the discount factor for buyers and lenders. The difference between discount factors demonstrates the potential gains from trade on the asset market.

Assets are indivisible Lucas trees. A type j asset generates a flow dividend $\delta_{t,j}$ in period t, where $j \in \{1, 2, ..., J\}$. Throughout this paper, I assume

[5] My results do rely on the risk-neutrality of buyers but can be generalized to the case where sellers and lenders are risk-averse.

that $J < \infty$. For all types, asset matures independently and randomly with a probability $\alpha \in (0, 1)$ in each period. $\{\delta_{t,j}\}_{j=1}^{J}$ are stochastic and follow a finite state Markov chain. For sake of simplicity, from now on I assume that type $j \geq 2$ assets have constant qualities.[6] The probability distribution of $\delta_{t+1,1}$ conditional on $\delta_{t,1}$ is written as $T^{\delta}(\cdot|\delta_{t,1})$ with a finite support ranging from $\underline{\delta}_1$ to $\bar{\delta}_1$. I label the quality of assets with an increasing order. That is, $0 < \underline{\delta}_1 \leq \delta_{t,1} \leq \bar{\delta}_1 < \delta_2 < ... < \delta_J$. The realization of $\{\delta_{t,j}\}_{j=1}^{J}$ are common knowledge for all agents. It captures the realism that the aggregate risk of assets are publicly revealed.

Sellers are heterogeneous in the type of assets they own, indexed by j. Each type j seller holds one unit of type j asset. Sellers with a matured or a sold asset will be replaced by an identical clone at the beginning of the next period. Therefore asset supply is fixed over time for any type j.[7] Let $\{M_j\}_j$ with $\sum_j M_j = \bar{M}$ denote the distribution of this fixed supply. Importantly, I assume that the quality type of a particular asset is only observable to its seller. In another word, a seller's type is private information. Therefore, the asset market suffers an adverse selection problem, and the extent of which varies over time as $\delta_{t,1}$ moves stochastically.

It will be clear after the complete description of the model that individual buyers are potentially subject to ex-post heterogeneity. I smooth the heterogeneity by assuming that individual buyers who belong to the same buyer family all pool their asset holdings together at the end of each period. As in Shi (2015) and Del Negro et al. (2017), this is a commonly used assumption in the literature to facilitate the aggregation and it allows me- to focus on the essence of the hypothesis without losing tractability. Notice that none of the price, the liquidity, and the run factors depend on the heterogeneity and the risk-sharing among buyers.

Buyers are present in both the asset market and the repo market. The budget of buyers' purchases on the asset market comes from two sources: internal funding generated from the dividends of asset holdings and borrowing from lenders via repo contracts. I assume each dollar of repo contract matures independently with probability β where $0 < \beta < \alpha$. Individual buyers are subject

[6]This is without lose of generality. Even though at an expanse of enlarging the state space, all my theoretical results hold without this assumption.

[7]All my analytical results remain with small modifications if I instead assume an exogenous variable asset supply, as long as agents in the model have perfect foresight for it. I believe that the supply side variation of assets is not the key driving force during the great recession since, without demand changes, the observed drop of asset supply will be inconsistent with the observed prices decrease.

to default risks that depend on the balance sheet condition of their family. If a default event happens, lenders associated with that buyer will take over the collateral portfolios and liquidate them on the market.

Each lender is endowed with $\bar{B}$ dollars when they are born. Lenders choose between a buyer-proposed repo contract and stochastic outside options. Similar to sellers, I assume the total measure of lenders is 1 for all t. Lenders are replaced by another lender in the next period if one of the following events happens: they choose the outside option; their repo contract is being repaid, and the buyer defaults. I assume lenders have heterogeneous utility for the same repo contract which is realized when they are born. Details will be specified in the later section. Figure 2 illustrates the repo market structure.

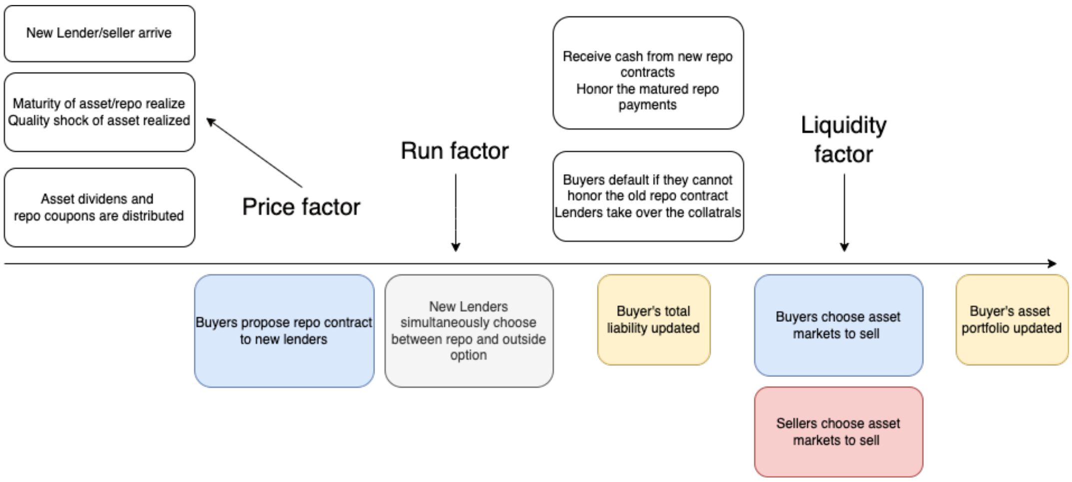

Figure 1: Timeline Within a Period

Figure 1 illustrates the timeline within each period. At the beginning of period t, the maturity shock and $\delta_{t,1}$ realize. Dividends are being distributed to asset owners. After that, new sellers and new lenders arrive. Observing these shocks, buyers propose repo contracts to lenders on the repo market. Conditional on the proposed repo offers and rational expectations on future asset prices and liquidities, lenders choose between their outside options and repo contract simultaneously. Buyers then get the cash from the repo market. They will honor the repo contract sold in the last period first and take whatever is left to the asset market[8]. Buyers and sellers then trade with each other.

I model the asset market by a competitive search framework à la Guerrieri

[8] As it will be clear in the later part, it is a dominated strategy that buyers carry a positive amount of cash to the next period without buying assets on the asset market.

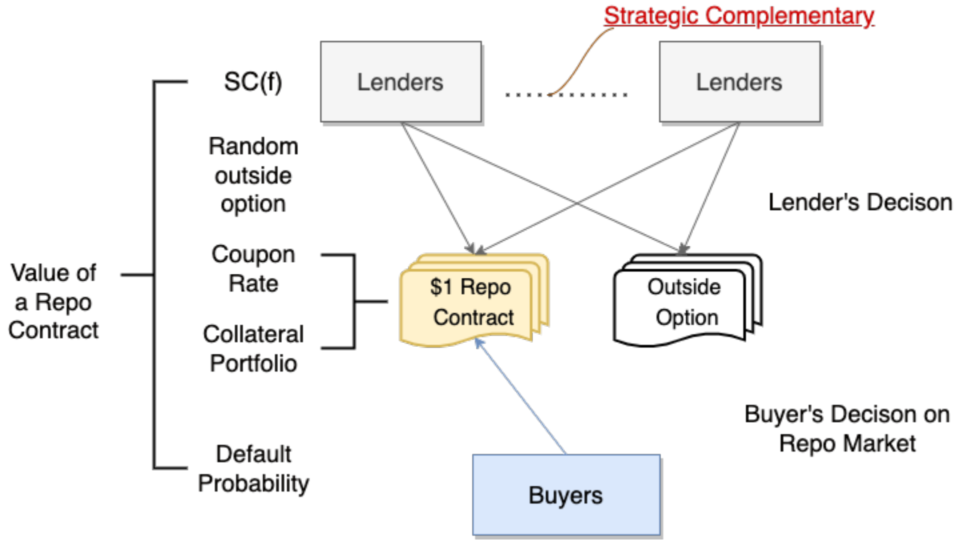

Figure 2: Repo Market Structure

and Shimer (2014). Figure 3 illustrates the asset market structure. Precisely, at each period t, a continuum of sub-markets identified by prices $p \in \mathbb{R}_+$ may open up. Both buyers and sellers can visit any such sub-market or an arbitrary set of sub-markets with their assets or cash. However, each unit of asset or dollar can only appear in one sub-market. Sellers are committed to selling their assets with a price of p in sub-market p if they are successfully matched with a buyer. Similarly, buyers undertake to buy the asset with a price of p in sub-market p if they are successfully matched with a seller.

Buyers and sellers have rational expectations for the likelihood of being matched in any sub-market p. Let $\Theta_t(p)$ denote the ratio between the amount of cash brought by buyers and the total amount of resources to purchase all assets in the sub-market p. I assume the probability that a seller can meet a buyer in sub-market p at time t is $\min\{\Theta_t(p), 1\}$[9]. That is, if $\Theta_t(p) \geq 1$, the total resources brought by buyers are more than enough to buy all assets in this sub-market. In such a case, every seller can successfully trade. If $\Theta_t(p) < 1$, every buyer can successfully trade and sellers are being rationed. On the contrary, the probability that a buyer can match with a seller is $\min\{\Theta_t(p)^{-1}, 1\}$. Buyers and sellers also have rational expectations for the (conditional) quality distribution of assets sold in each market p. Let $\Gamma_t(p) \equiv \{\gamma_{t,j}(p)\}_{j=1}^J \in \Delta^J$ in-

[9]This specification of the matching function highlights the role of the asymmetric information friction. As will be clear in the latter part of the paper, if there is no asymmetric information friction, all assets will be sold with probability one. This implies that the source of the liquidity issue is solely from the asymmetric information friction, not the search friction. With small modifications, all results can be generalized to commonly used matching functions in the literature.

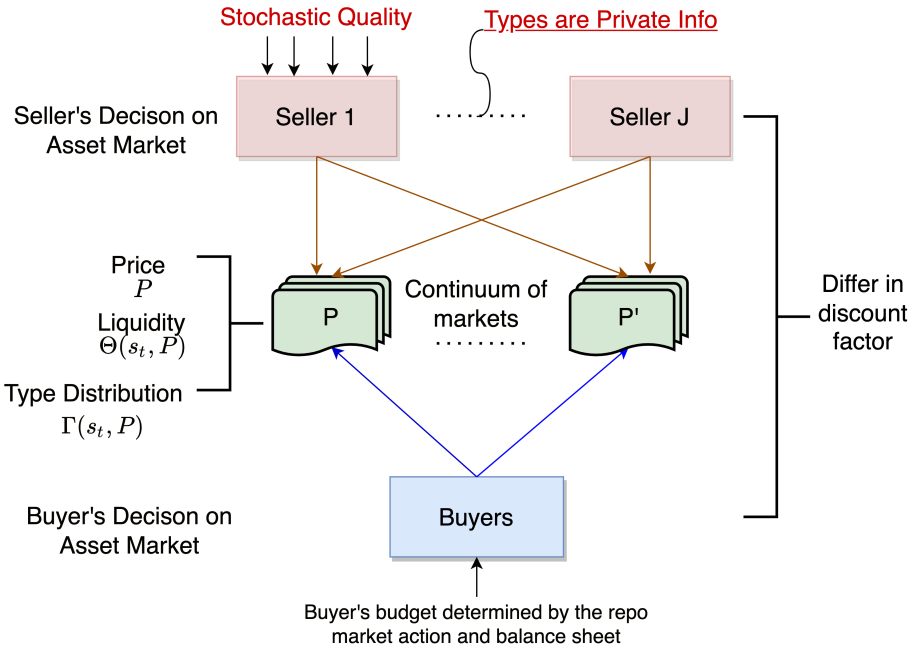

Figure 3: Asset Market Structure

dicates such distribution, where Δ^J is the J dimensional simplex. It is common knowledge that a successfully matched buyer can get a quality j asset in submarket p at period t with probability $\gamma_{t,j}(p)$. Notice that $\Gamma_t(p)$ contains both on-equilibrium path beliefs and off-equilibrium path beliefs. For convenience, I define the on equilibrium path (unconditional) cumulative distribution function of type j trade (over submarkets $p \in \mathbb{R}_+$) as $\Omega_{t,j}(p)$.

I will focus on the stationary Markov perfect equilibrium. Therefore, to save notations, I drop the time index t everywhere except for the state variable s_t. The aggregate state s_t represents the vector $\left(B_t, (K_{t,j})_j, \delta_{t,1}\right)$, where B_t is the total amount of repo contract owed by buyers in period t, $(K_{t,j})_j$ is the portfolio of asset holdings by buyers in period t, and $\delta_{t,1}$ is the random quality for the type 1 asset. I am ready to state agents' optimization problems.

3.2 Seller's Decision

In this part, I take the state s_t and its transition function as given. I will endogenize them in the general equilibrium part. Let $V^s_j(s_t)$ denote the value of type j seller at the beginning of period t. Taking the (equilibrium) market tightness function $\Theta(s_t, p)$ as given, the Bellman equation of seller's decision problem writes as follows

$$\frac{1}{1-\alpha}V_j^s(s_t) = \delta_{t,j} + \max_{p\in\mathbb{R}_+}\Big\{\min\{\Theta(s_t,p),1\}p$$
$$+\rho^l(1-\min\{\Theta(s_t,p),1\})\mathbb{E}_t\left[V_j^s(s_{t+1})\right]\Big\}. \tag{1}$$

The term $\frac{1}{1-\alpha}$ on the left-hand side takes care of the possibility that sellers are hit by the maturity shock. The first term on the right-hand side $\delta_{t,j}$ is the dividend obtained from holding the asset j. Notice that $\delta_{t,j} \equiv \delta_j$ for $j \geq 2$. Sellers optimally choose a sub-market indexed by p, in where she expect to meet a buyer with probability $\min\{\Theta(s_t,p),1\}$. When the trade is successful, the seller exit the market with the revenue p; when the trade does not happen, she keeps the asset and enters the next period. The conditional expectation is calculated according to the transition of aggregate state s_t.

3.3 Lender's Decision

Consider the decision problem of a lender when the aggregate state is s_t. I introduce several notations first. Suppose the buyer proposes a repo contract $\big(R,(k_j)_j\big)$, where R denotes the coupon spread over a lender's outside option and $(k_j)_j$ is the collateral portfolio. Given the rational expectation of asset prices and liquidities, I denote the liquidation value of a type j asset as $c_j(s_t)$:

$$c_j(s_t) = \int_{\mathbb{R}_+} p\cdot Pr\left(\min\{\Theta(s_t,p),1\}\right)\mathbf{d}\Omega_j(s_t,p)$$
$$+\left(1-\int_{\mathbb{R}_+} Pr\left(\min\{\Theta(s_t,p),1\}\right)\mathbf{d}\Omega_j(s_t,p)\right)\cdot \underline{\mathrm{v}}_j^s(s_t), \tag{2}$$

where $1-Pr\left(\min\{\Theta(s_t,p),1\}\right)$ is the probability that a lender has to fire sale the asset. If the fire sale event happens, $\underline{\mathrm{v}}_j^s(s_t)$ is the fire sale price. If the fire sale event does not happen, (2) says that the liquidation value of a type j asset is the average market price. The function $Pr(\cdot)$ is an increasing function of the market liquidity $\Theta(s_t,p)$. I will specify the exact functional form of it in the calibration section. I assume the fire sale price is the holding value of the asset by the agent who values it the least. In this case, that agent will be the seller. Therefore, I can recursively define

$$\underline{\mathrm{v}}_j^s(s_t) = (1-\alpha)\left\{\delta_{t,j}+\rho^l\mathbb{E}_t\left[\underline{\mathrm{v}}_j^s(s_{t+1})\right]\right\}. \tag{3}$$

Thus, the liquidation value for collateral portfolio $\left(k_j\right)_j$ is

$$C\left(s_t, \left(k_j\right)_j\right) = \sum_j c_j(s_t)k_j. \tag{4}$$

Suppose the buyer's cash balance is $Cash(s_t)$[10]. I denote the default probability of such buyer as $\pi\left(Cash(s_t)\right)$. $\pi(\cdot)$ is a decreasing function with a range of 0 to 1 and the exact functional form will be delayed to the calibration section. Let $v_j^l(s_t)$ be the expected discounted present value that a lender can recover from a unit of type j asset when a default event happens. And let $v_\pi^l(s_t)$ be the expected discounted present value that a lender suffers from the future default event. I can write

$$v_j^l(s_t) = \beta\pi \cdot c_j^l(s_t) + \rho^h(1-\alpha)(1-\beta)(1-\pi)\mathbb{E}_t\left[v_j^l(s_{t+1})\right]; \tag{5}$$

$$v_\pi^l(s_t) = -\beta\pi + \rho^h(1-\beta)(1-\pi)\mathbb{E}_t\left[v_\pi^l(s_{t+1})\right]. \tag{6}$$

To capture the strategic complementarity among lenders, I assume the lender's utility for the repo contract contains a term $\varphi \cdot SC(f)$.[11] φ is a constant and is the key parameter to be calibrated in the quantification exercise. f is the fraction of other lenders who choose the repo contract over the outside option and $SC(f)$ is an increasing function of f. The exact functional form is delayed to the later section. Similar to the idea in Morris and Shin (2001), I introduce incomplete information to refine the multi-equilibrium issue introduced by $SC(f)$. In particular, I assume each lender is born with a lender-specific heterogeneous utility for holding a repo contract per period: $u_{i,t}$. This random utility may come from lenders' heterogeneity in outside options, liquidation cost of the collaterals, and long-term relationships with the buyer, etc. It is common knowledge among lenders that $u_{i,t}$ has the structure

$$u_{i,t} = \mu_t + \epsilon_{i,t},$$

where μ_t is the aggregate component for all lenders. And $\epsilon_{i,t}$ is the idiosyncratic component. For tractability, I assume lenders (commonly) believe that $\mu_t \sim \mathcal{N}\left(0, \sigma_0^2\right)$ and $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2)$. Lenders observe the realization of $u_{i,t}$, but they cannot observe μ_t and $\epsilon_{i,t}$ separately. In my benchmark model, I consider

[10]This is an abuse of notation for simplicity. It will be clear in the next section that $Cash(\cdot)$ is a function of both s_t and the issuance of new repo contract NR. Since in the equilibrium, NR is also a function of s_t, I can substitute it out.

[11]Directly including this term into the lender's utility is a short-cut. My model is designed for quantifying the strategic complementarity, not explaining its source.

the case that the true realization of μ_t is always 0 for simplicity[12]. Therefore, μ_t is abbreviated from the aggregate state.

Thus, the difference of expected return between the repo contract and the outside option for a newborn lender is

$$V^l\left(s_t, R, (k_j)_j, f, u_{i,t}\right) = \frac{R + u_{i,t}}{1 - \rho^h(1-\beta)} + \varphi \cdot SC(f)$$
$$+ \rho^h \Big\{ \sum_j \mathbb{E}_t\big[v_j^l(s_{t+1})\big] k_j + \mathbb{E}_t\big[v_\pi^l(s_{t+1})\big] \Big\}. \tag{7}$$

For simplicity, I have assumed that the discounted sum of coupon payments is deducted immediately when contracting. Hence coupon payments are default-remote. As shown by the equation (7), a lender's utility is the sum of four parts: the expected protection from the collateral portfolio; the expected loss from the default event; the term represents strategic complementary; and the random utility part. The lender's decision problem is

$$\max\left\{V^l\left(s_t, R, (k_j)_j, f, u_{i,t}\right), 0\right\}. \tag{8}$$

This completes the description of an incomplete information simultaneous-move game among newborn lenders. To facilitate the further illustration, I denote this game as $\mathcal{G}\left(s_t, R, (k_j)_j\right)$.

3.4 Buyer's Decision

All buyer families are identical, so the aggregate state is the same as the state of a buyer family, which is denoted as s_t. Each buyer family contains a unit measure of individual buyers. This structure facilitates the aggregation and allows us to focus on the three factors without worrying about the intractability generated from buyers' heterogeneity. To achieve this, I assume at the end of each period, all individual buyers pool their balance sheets together and receive an equal share of both assets and liabilities in the next period. While balance sheets are fully insured among individual buyers, to provide incentives, consumption is distributed to each individual buyer according to their contribution to the total asset purchases. Suppose an individual buyer ℓ[13] has state

[12] Introducing aggregate uncertainty other than the stochastic quality of assets blurs my focus. The incomplete information about $u_{i,t}$ is only included for equilibrium refinement.

[13] The index ℓ will be abbreviated for simplicity wherever there is no confusion.

$\tilde{s}_t = \big((\tilde{K}_{t,j})_j\big)$, where $(\tilde{K}_{t,j})_j$ is the total asset stock purchased by this individual buyer. In period t, an individual buyer receives consumption $\omega \cdot \sum_j \tilde{K}_{t,j}\delta_{t,j}$ where $\omega \in (0,1]$ is fixed. Every buyer is obligated to repay the maturing repo liability βB_t. In return, she gets an amount of cash $(1-\omega)\cdot\sum_j K_{t,j}\delta_{t,j}$ for investment and a portfolio of assets $(\beta K_{t,j})_j$ as collaterals for new repo contract issuance. Due to the search friction on the asset market, at the end of a period, there may exist some cash used for investment but failed to trade. I assume the buyer family redistributes these cash back to individual buyers for immediate consumption in a lump sum way. Let us denote this term as $LT(s_t)$.

All relevant decisions are made by individual buyers. Unless otherwise indicated, "buyers" hereafter refers to individual buyers. Given an aggregate state s_t and a buyer's state $\tilde{s}_t$, each buyer decides the term and the amount of repo contract on the repo market and makes decisions on how much to invest and which submarket to visit on the asset market. The objective of a buyer is to maximize the discounted sum of her consumption. Mathematically, in each period t with state s_t and $\tilde{s}_t$, a buyer chooses a vector $\big\{NR, I, F, \big(R, (k_j)_j\big)\big\}$ where NR is the issuance of the new repo contract, I is the total amount invested on the asset market, $F(s_t, p)$ describes the cumulative distribution function of cash spent on each sub-markets, and $\big(R, (k_j)_j\big)$ indicates terms of the repo contract. The recursive optimization problem is

$$V^b(s_t, \tilde{s}_t) = \max \omega \cdot \sum_j \tilde{K}_{t,j}\delta_{t,j} + LT(s_t) + \rho^h \mathbb{E}_t\left[V^b(s_{t+1}, \tilde{s}_{t+1})\right]. \tag{9}$$

Buyers' state is subject to law of motion:

$$\begin{aligned}\tilde{K}_{t+1,j} = (1-\alpha)\Big\{&(1-\beta\cdot 1_d(s_t, NR))\tilde{K}_{t,j} \\ &+ I \cdot \int_{\mathbb{R}_+} \frac{\min\{\Theta^{-1}(s_t,p), 1\}}{p}\gamma_j(s_t,p)\mathbf{d}F(s_t,p)\Big\},\end{aligned} \tag{10}$$

where $1_d(s_t, NR)$ is the indicator function for the default event. The probability of default follows $\pi(Cash(s_t, NR))$. $Cash(\cdot)$ is the cash available for a buyer after coupon payment:

$$Cash(s_t, NR) = (1-\omega)\cdot\sum_j K_{t,j}\delta_{t,j} + \left(1 - \frac{R}{1-\rho^h(1-\beta)}\right)NR - \beta B_t. \tag{11}$$

Control variables are subject to resource constraints

$$0 \leq NR \leq f(s_t, R, (k_j)_j) \cdot \bar{B} \cdot (1 - 1_d(s_t, NR))\,; \tag{12}$$

$$0 \leq I \leq \max\{Cash(s_t, NR), 0\}\,, \tag{13}$$

where $f(s_t, R, (k_j)_j)$ is the equilibrium outcome of the game $\mathcal{G}(s_t, R, (k_j)_j)$. I also have constraints on the terms of the repo contract:

$$k_j \leq \frac{\beta K_{t,j}}{NR} \cdot (1 - 1_d(s_t, NR)) \quad \text{for all } j; \quad R \in [0,1]; \quad \text{and} \quad C\left(s_t, (k_j)_j\right) \geq 1.$$

The first constraint requires the promised collateral cannot exceed total available asset holdings and the third constraint requires the liquidation value of the collateral portfolio must weakly exceed the face value of a repo contract. The function $C\left(s_t, (k_j)_j\right)$ is defined in (4). The evolution of s_t is simply aggregation of $\tilde{B}_t$ and $(\tilde{K}_{t,j})_j$ across all buyers:

$$B_{t+1} = (1-\beta)B_t + \int_0^1 NR(\ell)\mathbf{d}\ell; \tag{14}$$

$$K_{t+1,j} = \int_0^1 \tilde{K}_{t+1,j}(\ell)\mathbf{d}\ell. \tag{15}$$

And the lump sum transfer $LT(s_t)$ is

$$LT(s_t) = \int_0^1 I(\ell) \cdot \left(1 - \int_{\mathbb{R}_+} \min\{\Theta^{-1}(s_t, p), 1\}\mathbf{d}F(p)\right)\mathbf{d}\ell. \tag{16}$$

Several observations are immediate from the buyer's problem. First of all, taking derivatives with respect to I and NR, I have that the upper bounds for both constraints (12) and (13) must be binding. The intuition is the following. Consider the trade-off for I first. The benefit from one more dollar of investment is clearly positive since it generates more future consumption. The cost of investment is not internalized by buyers because they are infinitesimal and equally share the balance sheet with the rest of their families. A similar idea goes for the new repo issuance NR. Issuing one more unit of repo contract can relax the constraint for I. From my previous discussion, this strictly benefits the buyer.

Secondly, the investment choice F is independent of $\tilde{s}_t$. This comes from the linearity of V^b in $\tilde{K}_{t,j}$. That is, the marginal benefit of having one more unit of type j asset depends only on the aggregate state. Thus, repo contracts $\big(R, (k_j)_j\big)$ are independent of $\tilde{s}_t$ since all lenders' decisions only depend on the aggregate state. Therefore, I conclude that the optimal choice $\big\{NR, I, F, \big(R, (k_j)_j\big)\big\}$ are all independent of individual buyers' state $\tilde{s}_t$.

As a consequence, the aggregate state s_{t+1} are independent of $\tilde{s}_t$ conditional on s_t. This observation greatly simplifies the optimization problem. Even though there is ex-post endogenous heterogeneity among buyers due to the search friction and default risk, I do not need to keep track of them.

For buyers who are hit by the default shock, clearly from the constraint (12), the new repo issuance is 0. The optimal repo contract has an arbitrary repo rate and zero collateral portfolio. The optimal investment is $(1-\omega)\cdot\sum_j K_{t,j}\delta_{t,j} - \beta B_t$. From now on, I focus on the buyers who do not default.

It is useful to define $v_j^b(s_t)$ as the discounted sum of dividends consumption by buyers from holding one unit of type j asset. The Bellman equation satisfies

$$v_j^b(s_t) = (1-\alpha)\Big\{\omega\delta_{t,j} + \rho^h(1-\pi(Cash(s_t, NR)))\mathbb{E}_t[v_j^b(s_{t+1})]\Big\}. \tag{17}$$

Thus, I can separate the buyer's optimization problem into two independent sub-problems.

Problem 1. *An individual buyer's repo issuing problem:*

$$\max_{R\in[0,1],(k_j)_j} (1 - \frac{R}{1-\rho^h(1-\beta)}) \cdot f(s_t, R, (k_j)_j),$$

subjects to

$$k_j \cdot f(s_t, R, (k_j)_j) \cdot \bar{B} \leq \beta K_{t,j};$$
$$C(s_t, (k_j)_j) \geq 1.$$

I denote the optimal policy of buyer's repo issuing problem as $R^*(s_t)$ and $(k_j^*(s_t))_j$. Let the total issuance of new repo contracts be $NR^*(s_t)$. I can simplify the law of motion for B_t in (14) as

$$B_{t+1} = (1-\beta)B_t + (1-\pi(s_t, NR^*)) \cdot NR^*(s_t). \tag{18}$$

Problem 2. *An individual buyer's asset investment problem:*

$$\lambda(s_t) = \max_{F(s_t,p)} \int_{\mathbb{R}_+} \sum_j \frac{\min\{\Theta^{-1}(s_t,p),1\} \cdot \rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{p} \gamma_j(s_t,p) \mathbf{d}F(s_t,p) \tag{19}$$

$\lambda(s_t)$ represents the rate of return when the aggregate state is s_t. Suppose the buyer brings a measure $\mathbf{d}F(s_t,p)$ of cash to the sub-market p. There are two possible results. With probability $\min\{\Theta^{-1}(s_t,p),1\}$, the buyer can successfully trade with a seller. Regardless of the seller's type, the buyer has to pay p for the asset. With probability $\gamma_j(s_t,p)$, the asset is of type j, and each type j asset generates a total discounted value $\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]$ for the buyer. I further assume the $\lambda(s_t)$ satisfy the following restriction

$$\lambda(s_t) \geq 1. \tag{20}$$

One rationalization is that buyers have access to a risk-free bond. Similar to the asset in my model, each buyer is able to consume an ω portion of it every period. To save notations, I take the shortcut (20) and restrict $\lambda(s_t)$ directly. Combining with the optimal investment for buyers who default, I obtain the total investment of a buyer family $I^*(s_t)$:

$$I^*(s_t) = \pi(s_t, NR^*) \cdot \max\left\{(1-\omega) \cdot \sum_j K_{t,j}\delta_{t,j} - \beta B_t, 0\right\} + (1 - \pi(s_t, NR^*))$$
$$\cdot \max\left\{(1-\omega) \cdot \sum_j K_{t,j}\delta_{t,j} - \beta B_t + (1 - \frac{R^*}{1-\rho^h(1-\beta)}) \cdot NR^*, 0\right\}. \tag{21}$$

Let the optimal policy function of buyer's investment problem be $F^*(s_t,p)$. I can simplify the law of motion for $K_{t,j}$:

$$K_{t+1,j} = (1-\alpha)\Big\{ (1 - \beta \cdot \pi(s_t, NR^*)) \cdot K_{t,j}$$
$$+ I^*(s_t) \cdot \int_{\mathbb{R}_+} \frac{\min\{\Theta^{-1}(s_t,p),1\}}{p} \gamma_j(s_t,p) \mathbf{d}F^*(s_t,p)\Big\} \tag{22}$$

4 Equilibrium

4.1 Partial Equilibrium on the Asset Market

In this section, I define and analyze the equilibrium. To simplify the exposition, I solve it backwardly. Let us consider the partial equilibrium of the asset market first. Recall that the aggregate state of the economy is $s_t = (B_t, (K_{t,j})_j, \delta_{t,1})$.

Definition 1. *Taking as given the aggregate state s_t, its transition function, and the rate of return $\lambda(s_t)$, the partial equilibrium on the asset market is a tuple of functions $(\{V_j^s(s_t)\}_j, \Theta(s_t, p), \Gamma(s_t, p), F^*(s_t, p))$ satisfying the following conditions: $\forall s_t$*

1. *Seller's Optimality: Given functions $\Theta(s_t, p)$ and $\Gamma(s_t, p)$, $V_j^s(s_t)$ solves (1), for all $j \in \{1, 2, ..., J\}$;*

2. *Buyer's Optimality: $\Theta(s_t, p)$, $\Gamma(s_t, p)$ and $\lambda(s_t)$ satisfies (19);*

3. *Equilibrium Beliefs: For all $j \in \{1, 2, ..., J\}$, s_t and p such that $\Theta(s_t, p) < \infty$ and $\gamma_j(s_t, p) > 0$, p solves the maximization problem on the right hand side of (1) for type j seller;*

4. *Active Markets: Given functions $\lambda(s_t)$, $\Theta(s_t, p)$ and $\Gamma(s_t, p)$, $F^*(s_t, p)$ solves the maximization problem on the right hand side of (19);*

5. *Consistency of Supplies with Beliefs: For every state s_t and any $j \in \mathcal{J}^*$, function $\Gamma(s_t, p)$ satisfies*

$$\frac{M_j}{\sum_{k \in J^*} M_k} = \int_{\mathbb{R}_+} \gamma_j(s_t, p) \mathbf{d} F^*(s_t, p); \tag{23}$$

   where

$$\mathcal{J}^* = \{j \in \{1, 2, ..., J\} | \Omega_j(s_t, p) > 0 \ \ \textit{for some } p > 0\},$$

   and

$$\Omega_j(s_t, p) = \int_0^p \gamma_j(s_t, p') \mathbf{d} F^*(s_t, p').$$

The equilibrium in Definition 1 is named a partial equilibrium since it takes s_t, its transition function and $\lambda(s_t)$ as given. I will endogenize these three objects in the general equilibrium part. The definition is a stochastic generalization of the equilibrium in Guerrieri and Shimer (2014). Conditions for sellers' and buyers' optimality are standard. For a buyer to get an asset j in submarket p with positive probability, I need two prerequisites: (1) the probability that the buyer can match with a seller is positive, which is equivalent to $\Theta(s_t, p) < \infty$ and (2) a type j asset is sold in submarket p, which translates to $\gamma_j(s_t, p) > 0$. Therefore, conditions for equilibrium beliefs states that regardless of a market p is active or not (on and off the equilibrium path), if a buyer expects that she can get a quality j asset with positive probability, it must be weakly optimal for type j seller to show up on sub-market p. Condition for active markets demonstrates the optimality of buyers' purchases on the asset market. The set $\mathcal{J}^*$ represents the set of assets that are actively traded.

Though the definition is straightforward, solving the equilibrium is not trivial. The main difficulty is that the equilibrium concept contains many high dimensional objects such as $\Theta(s_t, p)$ and $\Gamma(s_t, p)$. To overcome this problem, the partial equilibrium on the asset market is solved in several steps. The argument is a generalization of that in Guerrieri and Shimer (2014). I demonstrate a sketch of procedures here and leave the detailed discussion in Appendix *B*. To characterize the solution, I notice first that they can be reconstructed from solutions to a series of auxiliary problems $\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_J$.

Problem 3. *Taking as given the function $\lambda(s_t)$ and transition function of state s_t, the solution to problem $\mathcal{P}_j$ ($j \geq 1$) is a pair of functions (with slightly abuse of notations) $\left(V_j^s(\delta_{t,1}, \lambda(s_t)), p_j(s_t), \theta_j(s_t)\right)$ solving*

$$\frac{1}{1-\alpha} V_j^s(\delta_{t,1}, \lambda(s_t)) = \delta_{t,j} + \max_{p_j(s_t), \theta_j(s_t)} \Big\{ \min\{\theta_j(s_t), 1\} p_j(s_t) \\ + \rho^l (1 - \min\{\theta_j(s_t), 1\}) \mathbb{E}_t \left[V_j^s(\delta_{t+1,1}, \lambda(s_{t+1})) \right] \Big\}, \quad (24)$$

with constraints for every s_t

$$\lambda(s_t) \leq \frac{\min\{\theta_j^{-1}(s_t), 1\} \cdot \rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{p_j(s_t)}, \quad (25)$$

and for all $j' < j$, all s_t

$$V_{j'}^s(\delta_{t,1}, \lambda(s_t)) \geq \delta_{t,j'} + \min\{\theta_j(s_t), 1\} p_j(s_t) \\ + \rho^l (1 - \min\{\theta_j(s_t), 1\}) \mathbb{E}_t \left[V_{j'}^s(\delta_{t+1,1}, \lambda(s_{t+1})) \right]. \quad (26)$$

Here the constraint (26) for problem $\mathcal{P}_1$ is empty. And value functions $V^s_{j'}(\cdot)$ for $j' < j$ are defined in the solution to $\mathcal{P}_{j'}$ by mathematical induction.

The motivation for problem 3 is the following. There is a similarity between the intuitive criterion by Cho and Kreps (1987) and the equilibrium beliefs condition in definition 1. Moreover, it is well known that Spence's signaling game has a unique equilibrium outcome that survives the intuitive criterion – the separating equilibrium. It will not be surprising that better sellers in my model signal their qualities by posting higher prices and the equilibrium is separating, too. This observation (guess) greatly simplifies the problem by reducing the infinite-dimensional objects, conditional on s_t, $\Theta(s_t, p)$ and $\Gamma(s_t, p)$ to real numbers $(p_j(s_t), \theta_j(s_t))_j$ on the equilibrium path. Another simplification is that, instead of solving both the buyer's optimization and seller's optimization problems, I take the buyer's reservation value as given and let sellers make choices. It is the *market utility approach*[14] commonly used in this type of model.

To solve the problem $\mathcal{P}_j$, I proceed with several lemmas, which are formally stated and proved in Appendix B. Lemma B.1 proves the monotonicity of $V^s_j(\cdot)$ with respect to j conditional on state s_t. The idea is that whenever a policy $p_{j'}(s_t), \theta_{j'}(s_t)$ is feasible to problem $\mathcal{P}_{j'}$, the same policy must also be feasible for $\mathcal{P}_j$ with $j > j'$. problem 3, implies important single crossing conditions among the values $V^s_j(\cdot)$. That is, higher types would put more weight on asset prices than liquidities when considering the trade-off between these two factors.

Lemma B.3 confirms that for all j, the constraint (25) is binding at every state s_t. To see it, for a contradiction, suppose (25) does not bind. When $j = 1$, the statement is clear since a seller can always increase the price and be better off without influencing anything else. When $j > 1$, however, I need to worry about constraints (26) (hereafter I call them IC constraints). The strategy is to increase $p_j(s_t)$ and decrease $\theta_j(s_t)$ at the same time so that (25) is binding while (26) are still satisfied. This is possible thanks to the single crossing condition implied from lemma B.1. Moreover, the deviating strategy increases seller j's value. Hence I have found a contradiction.

Next, in lemma B.5, I demonstrate that IC constraints are binding between adjacent types, i.e. j and $j-1$ for all $j > 1$, and are slack otherwise. Let us take the example of the problem $\mathcal{P}_2$ and suppose that IC is slack. Thus, both problem $\mathcal{P}_1$ and problem $\mathcal{P}_2$ have no IC constraints. Intuitively, there is no reason

[14]See Wright et al. (2019) page 5 for application of such methodology.

for sellers, regardless of their types, to lower down the probability of trade. Lemma B.4 confirms this intuition and proves that, under this condition, optimal choices of both $\theta_1(s_t)$ and $\theta_2(s_t)$ are weakly larger than 1. However, since constraint (25) are binding which implies $p_1(s_t) < p_2(s_t)$, the IC constraint between type 2 and type 1 sellers must be violated: there is strict incentive for type 1 seller to mimic the type 2 seller for higher prices without lowering down the trading probability.

Lemma B.6 exhibits the existence and uniqueness of solutions to problem $\mathcal{P}_j$. This follows from a simple mathematical induction. Since constraint (25) is binding in $\mathcal{P}_1$. After substituting it back to the objective function and recalling that $\theta_1(s_t) \geq 1$, a standard contraction mapping argument shows the existence and uniqueness of $V_1^s(\cdot)$. From the induction hypothesis, taking as given $V_{j-1}^s(\cdot)$, I can solve $p_j(s_t)$ and $\theta_j(s_t)$ as functions of $\lambda(s_t)$, $V_{j-1}^s(\cdot)$ and $v_j^b(\cdot)$ from binding constraints (25) and (26). Substituting these back to the objective function, again, I am ready to apply the standard contraction mapping routine. This proves the existence and uniqueness.

The last step is to confirm the equivalence between solutions to auxiliary problems and the original partial equilibrium on the asset market. On the one hand, lemma B.7 argues that solutions to problems $\{\mathcal{P}_j\}_j$, after some modifications, can serve as a partial equilibrium in definition 1. Notice that $\theta_j(s_t)$ and $p_j(s_t)$ are prices and liquidities for active markets (on the equilibrium path), I still need to construct the market tightness function for inactive markets from equilibrium beliefs condition. The idea is to use the equilibrium belief condition in definition 1. Thus, combined with lemma B.6, lemma B.7 shows the existence of partial equilibrium on the asset market. On the other hand, I provide the reverse result in lemma B.8 such that from every partial equilibrium in definition 1, I can construct solutions for problems $\{\mathcal{P}_j\}_j$. This result establishes the uniqueness of the partial equilibrium on the asset market.

I summarize the main findings with the following proposition. For all s_t, let us define $\bar{\lambda}(s_t)$ as

$$\bar{\lambda}(s_t) \equiv \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{(1-\alpha)\rho^l \mathbb{E}_t\left[\delta_{t+1,1} + \frac{\rho^h \mathbb{E}_{t+1}[v_1^b(s_{t+2})]}{\lambda(s_{t+1})}\right]};$$

and $\underline{\lambda}(s_t) \equiv 1$.

Proposition 1. *Taking as given the aggregate state s_t and assume its transition function satisfies Feller property. Suppose functions $\lambda(s_t)$ is continuous in s_t. There ex-*

ists a partial equilibrium on asset market. Moreover, if $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$ for all states s_t, the equilibrium is unique. Suppose $\big(p_j(s_t), \theta_j(s_t), V_j^s(\delta_{t,1}, \lambda(s_t))\big)$ is the solution to problems $\{\mathcal{P}_j\}_j$, for all states s_t, the equilibrium satisfy

1. *Separating markets: for all j, $\gamma_j(s_t, p_j(s_t)) = 1$;*
2. *No distortion at the bottom: one of the following two cases are true*
   (i) $\Theta(s_t, p_1(s_t)) = \theta_1(s_t) \geq 1$ and $0 \leq \Theta(s_t, p_j(s_t)) = \theta_j(s_t) < 1$ for $j > 1$
   (ii) $\Theta(s_t, p_1(s_t)) = \theta_1(s_t) < 1$ and $\Theta(s_t, p_j(s_t)) = \theta_j(s_t) = 0$ for $j > 1$
3. *Seller's value increasing in types: for all j, $V_j^s(\delta_{t,1}, \lambda(s_t)) = V_j^s(s_t)$ and $V_1^s(s_t) < V_2^s(s_t) <, ..., < V_J^s(s_t)$;*
4. *Decreasing liquidities and increasing prices in types: $\theta_1(s_t) > \theta_2(s_t) \geq, ..., \geq \theta_J(s_t)$ and $p_1(s_t) < p_2(s_t) <, ..., < p_J(s_t)$.*

On the equilibrium path, prices $p_j(s_t)$ and liquidity $\theta_j(s_t)$ are determined by the following equation system.

$$p_j(s_t) = \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)}, \text{ for all } j \in \{1, 2, ..., J\} \tag{27}$$

$$\theta_j(s_t) = \min\{\theta_{j-1}(s_t), 1\} \cdot \frac{p_{j-1}(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]}{p_j(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]} \text{ for all } j \geq 2, \tag{28}$$

with $\theta_1(s_t) \geq 1$ when $\lambda(s_t) > \underline{\lambda}(s_t)$ and $\theta_j(s_t) = 0$ for all $j \geq 2$ if $\lambda(s_t) = \bar{\lambda}(s_t)$. The complete description of $\Theta(s_t, \cdot)$ and $\Gamma(s_t, \cdot)$ with proofs are presented in appendix B. To further verify that the model can capture the intuition of my hypothesis, I investigate comparative statics properties of the partial equilibrium on the asset market. I verify that when the "market condition" leans towards buyers, which is captured by a higher $\lambda(s_t)$ (in a single state s_t), both the equilibrium price and the liquidity drops. Results are summarized in the proposition below and the proof is delegated to the appendix B.

Proposition 2. *Fix a transition function of s_t and assume it satisfies Feller property. Fix functions $\lambda(s_t)$ and suppose they are continuous in s_t. Suppose $\big(\{V_j^s(s_t)\}_j, \Theta(s_t, p), \Gamma(s_t, p), F(s_t, p)\big)$ is a partial equilibrium associated with function $\lambda(s_t)$. Suppose also $\big(\tilde{V}^b(s_t), \{\tilde{V}_j^s(s_t)\}_j, \tilde{\Theta}(s_t, p), \tilde{\Gamma}(s_t, p), \tilde{F}(s_t, p)\big)$ is a partial equilibrium associated with function $\tilde{\lambda}(s_t)$. Given any state s_t and j, if $\bar{\lambda}(s_t) > \tilde{\lambda}(s_t) > \lambda(s_t) > \underline{\lambda}(s_t)$ and $\bar{\lambda}(s_t') > \tilde{\lambda}(s_t') = \lambda(s_t') > \underline{\lambda}(s_t)$ for all $s_t' \neq s_t$, the equilibrium price satisfies $\tilde{p}_j(s_t) < p_j(s_t)$ and liquidity satisfies $\tilde{\theta}_j(s_t) \leq \theta_j(s_t)$.*

4.2 Partial Equilibrium on the Repo Market

In this subsection, I define and characterize the equilibrium among lenders in the repo market. For any repo contract and aggregate state s_t, I say a lender who observes $u_{i,t}$ employ a threshold strategy if there exists a threshold function $\kappa^*\left(s_t,\left(R,(k_j)_j\right)\right)$ such that the lender

$$\begin{cases} \text{Chooses Repo Contract} & \text{if } u_{i,t} \geq \kappa^*\left(s_t,\left(R,(k_j)_j\right)\right) \\ \text{Chooses the Outside Option} & \quad \text{Otherwise} \end{cases}.$$

For simplicity, I focus on the symmetric threshold equilibrium.

Definition 2. *Taking as given a repo contract $\left(R,(k_j)_j\right)$; the aggregate state s_t and its transition function; and an asset market partial equilibrium $\Theta(s_t,p)$ and $\Gamma(s_t,p)$, a symmetric threshold equilibrium on the repo market is a threshold strategy employed by all lenders that survives IDSDS (iterative deletion of dominated strategies) of $\mathcal{G}(s_t,R,(k_j)_j)$.*

By this definition, since I assumed $\mu_t \equiv 0$ and the equilibrium threshold is $\kappa^*\left(s_t,\left(R,(k_j)_j\right)\right)$, the fraction of lenders taking the repo contract is

$$f\left(s_t,\left(R,(k_j)_j\right)\right) = 1-\Phi\left(\frac{\kappa^*\left(s_t,\left(R,(k_j)_j\right)\right)}{\sigma}\right). \tag{29}$$

where $\Phi(\cdot)$ is the *c.d.f* for the standard normal distribution. Let $h(s_t)$ be the haircut for newly issued repo contracts and $H(s_t)$ denotes the aggregate average haircut rate in the economy. They are defined by

$$h(s_t) = 1-\frac{f^*\left(s_t,(R^*,k_j^*)\right)\cdot\bar{B}}{\sum_j p_j(s_t)k_j^*(s_t)}. \tag{30}$$

$$H(s_t) = 1-\frac{B(s_{t+1})}{\sum_j p_j(s_t)k_j^*(s_t)}. \tag{31}$$

If parameters satisfy the following condition, It is confirmed that the partial equilibrium on the repo market is unique.

Assumption 1. *Suppose $SC(f)$ is differentiable with respect to f and*

$$\frac{\sigma}{\sigma_0^2+\sigma^2} \leq \frac{\sqrt{2\pi}}{1-\rho^h(1-\beta)}\cdot\inf_{f\in[0,1]}\left\{\frac{1}{\varphi\cdot SC'(f)}\right\}. \tag{32}$$

Assumption 1 requires the relative noise of the idiosyncratic component of $u_{i,t}$ is small enough compared with the noise of the aggregate component. The intuition is the following. A smaller *LHS* implies that, if a lender observes a higher realization of $u_{i,t}$, his posterior belief about the aggregate component μ_t increases sufficiently such that his private utility indeed ranks lower among all lenders. Thus, the likelihood that other lenders will run on the buyer becomes very low. The aggregate component μ_t essentially coordinated lenders towards an equilibrium without the run. This coordination is a force that works against the equilibrium multiplicity. Not surprisingly, the upper bound on the *RHS* depends on the level of strategic complementarity. For example, consider an extreme case that $\varphi \to +\infty$. In this case, if f drops a little bit, since $SC'(f) > 0$, the utility for a lender choosing repo drops to $-\infty$. As a consequence, there does not exist any parameter pair σ and σ_0 that satisfies the assumption 1.

A sketch of the argument is the following. Notice that $u_{i,t}$ is normally distributed. Hence, there exist, lenders, who will always reject the repo contract and who will always accept the repo contract. I say such lenders are in the dominance region. Since lenders are strategically complementary to each other, I expect that beginning from the two dominance regions, no matter how small they are, the contagion effect iteratively enlarges the two regions until they agree with each other. As a consequence, similar to Morris and Shin (2001), the only symmetric strategy surviving the iterated deletion of strictly dominated strategies is characterized by a threshold value. I delegate the detailed proof of this result to the appendix C. I summarize the main findings with the following propositions.

Proposition 3. *Suppose the assumption 1 holds. For any repo contract $\left(R, (k_j)_j\right)$, taking as given the aggregate state s_t and its transition function, an asset market partial equilibrium $\Theta(s_t, p)$ and $\Gamma(s_t, p)$, there exists an unique partial equilibrium on the repo market.*

The equilibrium threshold function $\kappa^* \left(s_t, \left(R, (k_j)_j\right)\right)$ solves the following equation

$$0 = \frac{R + \kappa^*}{1 - (1-\beta)\rho^h} + \int_{\mathbb{R}} \varphi \cdot SC\left(\tilde{f}(x, \kappa^*)\right) \mathbf{d}\Phi(x) + \rho^h \Big\{ \sum_j \mathbb{E}_t\left[v^l_j(s_{t+1})\right] k_j + \mathbb{E}_t\left[v^l_\pi(s_{t+1})\right] \Big\}, \tag{33}$$

where $\tilde{f}(x,\kappa^*)$ is defined by

$$\tilde{f}(x,\kappa^*) = 1 - \Phi\left(\frac{\kappa^* - x\sqrt{\frac{\sigma^2\cdot\sigma_0^2}{\sigma^2+\sigma_0^2}} - \frac{\sigma_0^2\cdot\kappa^*}{\sigma^2+\sigma_0^2}}{\sigma}\right), \tag{34}$$

and $\Phi(x)$ denotes the CDF of standard Normal distribution.

4.3 General Equilibrium

Finally, I define the general equilibrium in the economy. It is a Markov Perfect Equilibrium with the payoff-relevant state s_t.

Definition 3. *General equilibrium is a tuple of functions*

$$\left\{\{V_j^s(s_t)\}_j, \Theta(s_t,p), \Gamma(s_t,p), F^*(s_t,p), \lambda(s_t), \kappa^*(\cdot), \left(R^*(s_t),(k_j^*(s_t))_j\right)\right\}$$

and a transition function $T(s_{t+1}|s_t)$ for aggregate states such that, at every state s_t:

1. *Given $T(s_{t+1}|s_t)$ and $\lambda(s_t)$, $\left(\{V_j^s(s_t)\}_j, \Theta(s_t,p), \Gamma(s_t,p), F^*(s_t,p)\right)$ is a partial equilibrium on asset market in definition 1;*
2. *Given $T(s_{t+1}|s_t)$, $\Theta(s_t,p)$, $\Gamma(s_t,p)$ and $\left(R^*(s_t),(k_j^*(s_t))_j\right)$, $\kappa^*(\cdot)$ is a partial equilibrium on the repo market as in definition 2;*
3. *Given $f(\cdot)$ implied from $\kappa^*(\cdot)$ defined in (29), $\left(R^*(s_t),(k_j^*(s_t))_j\right)$ solves the optimal repo contract problem in problem 1; and given $\Theta(s_t,p)$ and $\Gamma(s_t,p)$, $\lambda(s_t)$ coincides with the optimal value of buyer's asset investment problem in problem 2;*
4. *$Cash(s_t,NR^*)$ defined in (11) satisfy*

$$\sum_j p_j(s_t)\theta_j(s_t)M_j = Cash(s_t,NR^*); \tag{35}$$

5. *$T(s_{t+1}|s_t)$ coincides with law of motions (18) and (22).*

5 Calibration

5.1 Functional Forms

There are three functional forms that need to be specified for the calibration: the default probability $\pi(Cash)$ as a function of $Cash$, the fire sale probability $Pr(\theta)$ as a function of the market liquidity θ, and the strategic complementarity term in lender's utility $SC(f)$ as a function of f, which is the fraction of lenders who choose the repo contract.

Let $Cash$ be a buyer's available cash balance. Suppose the notation $[y]^+$ represents the non-negative part of y and $[y]^-$ refers to the non-positive part of y. I assume

$$\pi(Cash) = \max\left\{\epsilon - 1 + e^{\eta\cdot[Cash]^+} + 1 - e^{-\eta\cdot[Cash]^-}, 0\right\},$$

where $\epsilon > 0$ and $\eta < 0$. Clearly, the function $\pi(Cash)$ is S-shaped, continuously differentiable, and decreasing in $Cash$. Both parameters have immediate economic implications: $\epsilon = \pi(0)$ anchors the level of π and η controls the derivative of π with respect to $Cash$. Notice that when η is very small, the function π is close to a linear function around 0 with intercept ϵ and slope η. One micro-foundations for this functional form is the following. Since my model only focuses on the repo liability and its associated collateral assets, $Cash < 0$ does not necessarily imply buyers, large dealer banks, in reality, will default. They can find other resources outside the repo market. For example, they may get revenues from financial services, derivative trading, and even commercial banking services, etc. However, it is without any doubt that the larger the cash gap is, the harder for buyers to get it from other places. If the arrival of external resources of cash follows the Poisson process and the rate is linear in the cash gap, the default probability will be the one I have illustrated above.

Next, suppose the market liquidity for an asset is θ. I assume a lender has to fire sale the asset with the probability

$$Pr(\theta) = 1 - \frac{\nu_3}{\{1 + \nu_4 \cdot \exp(-\nu_2(\theta - \nu_1))\}^{1/\nu_4}},$$

where $\nu_j > 0$ for all $j = 1, ..., 4$. This is the generalized logistic function, which is S-shaped, continuously differentiable and decreasing in θ. ν_1 controls inflection point of Pr, v_2 represents the upper bound of the slope, v_3 illustrates the asymptotic upper bound of Pr; and v_4 is a shape parameter.

Finally, I specify the $SC(f)$ as follows

$$SC(f) = \ln\left(\frac{f}{\bar{f}}\right) \cdot 1(f \geq \underline{f}) + 1(f < \underline{f}) \cdot \left\{\frac{\bar{f}}{\underline{f}} \cdot f + \ln\left(\underline{f}/\bar{f}\right) - \bar{f}\right\},$$

where $\underline{f}$ and $\bar{f}$ are two parameters such that $0 < \underline{f} < \bar{f} < 1$ and $\underline{f}$ is small enough. The second term above is introduced to eliminate the possible technical difficulties due to the divergence of $ln(\cdot)$ when f is small while maintaining the continuous differentiability of the function $SC(f)$. The economic implication of the function is the following. A lender's disutility of choosing repo contract rises exponentially when a smaller fraction of other lenders accept the repo offer, compared with a benchmark fraction $\bar{f}$. This is consistent with the literature that emphasizes the contagion effect/network externality among lenders during a run.

5.2 Directly Assigned Parameters

Parameters are divided into two groups. A group that clearly corresponds to an observable measure or parameters that can be normalized without loss of generality. And another group of "deep parameters" that need to be estimated carefully. In this subsection, I consider the first group of parameters. The length of the period in the model is one week.

Table 1 reports the value and calibration targets of directly assigned parameters. The discount factor $\rho^h = 0.98$ is nonstandard.[15] The reason for choosing such a low discount factor is the following. As argued in Stanton and Wallace (2011), ABX prices[16] in 2007-2009 are not consistent with any reasonable mortgage default and recovery rate. Consider a back-of-envelop calculation of the ABX.HE CDS. Let P be the fair price of the default insurance, EL be the per dollar expected loss of an average subprime mortgage. I have

$$EL \simeq \frac{D(1 - Rec)}{1 - \rho^h} + 1 - \left(\rho^h\right)^L,$$

[15] A standard weekly discount factor calculated from the LIBOR rate should be around 0.999.

[16] It is a synthetic tradable index referencing the CDS of a basket of subprime mortgage-backed securities constructed by IHS Markit.

where D represents the periodic default probability, Rec denotes the recovery rate of default loans, and L is the delay of pre-payment (in weeks) due to the difficulty of re-financing. Suppose the size of all subordination tranches is S. I obtain

$$1 - P = \max\left\{\frac{EL - S}{1 - S}, 0\right\}. \tag{36}$$

Given a standard ρ^h value, Stanton and Wallace (2011) find that the P calculated from ABX index implies $EL \geq 1$. That is, the market expects the discounted sum of losses exceeds the principal. They use this calculation as evidence that the ABX index is not correctly priced. I view it in a different way. Conditional on that the market expect a reasonable default rate D and historical recovery rate Rec, an extremely low P can come about if the discount factor ρ^h is very low[17]. The low ρ^h captures the potential capital constraints and the holding cost, etc. of buyers. None of these factors are endogenous in my model, so I use a low ρ^h as a shortcut way to control them. Consistent with my view, recent literature like Saleuddin and Jansson (2021) econometrically confirms that fundamental-driven components can explain the ABX index drop in 2007-2009.

Other parameters are standard. All calculations are straightforward except for $\bar{B}$. Notice that $\bar{f}$ is chosen to be 0.5 for convenience. I assume that before the crisis, the fraction of lenders accepting repo contracts is 0.5. So given the *Total Liability* implied from the initial state (representing the pre-crisis level) of the simulation, I can calculate $\bar{B} = \beta \times \text{Total Liability} \div \frac{1}{2} = 1763.85$. The construction of the initial state will be elaborated in section 6.1. Also, notice that $\omega \cdot \delta_2$ is not calibrated separately since they always show together in my model.

5.3 Three Factors

Now turn to the "deep parameters" that discipline the price factor, the liquidity factor, and the run factor. I start from the price factor. The key parameters are the support and the transition matrix of the $\delta_{t,1}$ process. They reflect the quality change of the low type assets. As illustrated in (36), the change of the ABX index indicates the present value changes of expected write-downs and

[17] For example, consider the following parameters. Suppose that $D = 0.25$, $Rec = 0.4$, $P = 0.3$, $S = 0.38$, and $L = 52$. The implied $\rho^h = \sqrt[52]{0.336} \simeq 0.98$.

Table 1: Directly Assigned Parameters

Parameter	Target	Value	Source
ρ^h	Exogenously Assigned	0.98	irregularity in ABX
β	Average length of repo contract: 6 weeks	1/6	ICMA survey
α	Average length of RMBS: 216 weeks	1/216	Malkhozov et al. (2016)
J	Loss distribution of RMBS: bi-modal	2	Ospina and Uhlig (2018)
$\omega \cdot \delta_2$	Normalization	1	none
$\bar{B}$	Trading volume of RMBS before crisis	1763.85	Ospina and Uhlig (2018)
$\bar{f}$	Normalization	0.5	For convenience
$\underline{f}$	Small enough	0.2	none

shortfalls of the underlying RMBS. Therefore, I can use it to calibrate the $\delta_{t,1}$ process in my model. ABX indices for all ratings and all RMBS baskets are highly correlated and behave similarly in 2006-2009. Therefore, I choose the indices for ABX.HE-2007-01 with AAA credit rating without loss of generality. The support and the transition matrix for $\delta_{t,1}$ is estimated with the following procedure. First of all, I grid the range of the index observations evenly into 10 discrete points, denoted as $\delta_{t,1}(k)$ for $k = 1, ..., 10$. This will be the support for $\delta_{t,1}$. In the next step, each observation is approximated by the closest grid. Conditional on a grid k, I can calculate the empirical frequency $F(k, k')$ that the observation transit from the grid k to other grids k'. Using $F(k, k')$ for all k, I can construct the transition matrix of $\delta_{t+1,1}|\delta_{t,1}$. Since most observations are in the crisis time, there are some grids that only go down. This may bias the result in my model since it implies a multiplicity of recurrent families of the Markov process, making some of the $\delta_{t,1}$ drop being permanent. I make one modification on $F(k, k')$ to fix this issue. For those grid $k \notin \{1, 10\}$ and only has downward observations, I assume $\mathbb{E}(\delta_{t+1,1}|\delta_{t,1}(k)) = \delta_{t,1}(k)$. The idea is inclined to the efficient market hypothesis. Table 3 reports the final result.

All remaining parameters are calibrated using the standard simulated method of moments (SMM). In the following, I discuss the targets used to identify these parameters first and then elaborate on the procedures of SMM in the next section. Consider the parameters related to the liquidity factor. The most important parameters are the distribution of asset supplies: $(M_j)_j$. I target the fact in Ospina and Uhlig (2018) that for assets issued up until 2008, nearly 80% of all AAA rated RMBS suffers losses less than 5%. That is, among asset stocks hold by buyers up to 2008, 80% of them are of high type. $\{\rho^l, \nu_1, ..., \nu_4\}$ are estimated by matching the $H(s_t)$ in my model with the haircut path[18] depicted in Gorton and Metrick (2012). The intuition is that ρ^l[19] determines the seller's holding value of the asset, which is assumed to be the fire-sale price when lenders liquidate collaterals. $\nu_1, ..., \nu_4$ control the probability of a fire sale event, therefore having a direct impact on the haircut rates. $\{\eta, \epsilon\}$ are important for the default probability of buyers. Following the literature, I use the LIB-OIS rate as an approximation of the default risk.

Finally, I consider parameters related to the run factor. Unfortunately, none of $\{\varphi, \sigma_0, \sigma\}$ have measurable counterparts in the data. I calibrate them indirectly. The intuition is the following. From some manipulations of the equation (33), the equilibrium repo rate can be represented by the sum of four terms:

$$\frac{-R}{1-\rho^h(1-\beta)} = \rho^h \cdot \Big\{ \sum_j \mathbb{E}_t\big[v_j^l(s_{t+1})\big]k_j^* + \mathbb{E}_t\big[v_\pi^l(s_{t+1})\big] \Big\}$$
$$+ \varphi \cdot \Upsilon(\frac{\sigma}{\sigma_0}, f_t^*) + \sigma \cdot \Phi^{-1}(1 - f_t^*), \tag{37}$$

where $\Upsilon(\frac{\sigma}{\sigma_0}, f_t^*)$ is a known function. The first term connects to the over-collateralization of a repo contract. It negatively affects the repo rate and is disciplined by the haircut path in the target. The second term relates to the default risk of buyers and is positively correlated with the repo rate. This term is accounted for by targeting the LIB-OIS rate. The third term reflects the strategic complementarity among lenders and the fourth term is about the heterogeneity of lenders' private utilities. With the simulated path for s_t, $k_{t,j}^*$ and f_t^*, I target the average repo rate associated with RMBS collateral documented in Gorton and Metrick (2012) in three periods: the first half of 2007, the second half of 2007 and 2008. From equation (37), these targets should contain information of $\{\varphi, \sigma_0, \sigma\}$. Since the simulation is built on the uniqueness of the repo market equilibrium , during the estimation, σ_0 and σ are restricted to

[18] Digitalized by the WebPlotDigitizer and reproduced by the author.

[19] Similar with ρ^h, the discount factor of sellers may also contain other terms like the asset holding cost or regulatory burdens.

satisfy the assumption 1. The calibrated values are reported in table 2. Table 4 and table 5 illustrate the local sensitivity of the targeted moments with respect to the benchmark calibration.

Table 2: Jointly Calibrated Parameters

Parameter	Targets	Value	Source
M_1	Loss Distribution	0.1260	Ospina and Uhlig (2018)
M_2	of AAA RMBS	1.2095	
ρ^l		0.9613	
ν_1	haircut	0.0295	
ν_2	path	252.4297	Gorton and Metrick (2012)
ν_3		0.9244	
ν_4		5.90e-8	
η	LIB-OIS spread	-9.5246e-6	Bloomberg
ϵ		0.000347	
φ	Average repo	298.686	
σ	rate in three	0.002808	Gorton and Metrick (2012)
σ_0	periods	0.5720	

Table 3: The Support and Transition Matrix of $\delta_{t,1}$

	0.299	0.377	0.455	0.533	0.611	0.689	0.766	0.844	0.922	1.000
0.299	0.715	0.285	0	0	0	0	0	0	0	0
0.377	0.111	0.778	0.111	0	0	0	0	0	0	0
0.455	0	0.025	0.950	0.025	0	0	0	0	0	0
0.533	0	0	0.070	0.861	0.070	0	0	0	0	0
0.611	0	0	0	0.044	0.913	0.044	0	0	0	0
0.689	0	0	0	0	0.067	0.866	0.067	0	0	0
0.766	0	0	0	0	0	0.057	0.885	0.057	0	0
0.844	0	0	0	0	0	0	0.018	0.964	0.018	0
0.922	0	0	0	0	0	0	0	0.018	0.964	0.018
1.000	0	0	0	0	0	0	0	0	0.01	0.99

Table 4: Percentage Changes of Target Moments When Benchmark Parameters are Perturbed +1%

Moments\Parameters	ρ^l	ν_1	ν_2	ν_3	ν_4	η	ϵ	φ	σ	σ_0
Mean Hair Cut 2007	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	-0.01%	-12.98%	0.00%	0.10%
Mean Hair Cut 2008 Q1	0.00%	0.00%	0.00%	0.00%	-0.02%	0.46%	-0.14%	-2.18%	-0.02%	-1.95%
Mean Hair Cut 2008 Q2	0.00%	0.00%	0.00%	0.00%	0.01%	4.41%	-0.45%	-2.30%	0.01%	0.35%
Mean Hair Cut 2008 Q3Q4	0.00%	0.00%	0.00%	0.00%	0.00%	0.40%	-0.03%	-0.30%	0.00%	0.02%
LIB-OIS 2007 Q1Q2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
LIB-OIS 2007 Q3Q4	0.02%	-0.03%	0.01%	0.59%	0.41%	0.11%	-0.06%	1.49%	-0.02%	4.51%
LIB-OIS 2008	0.03%	-0.04%	0.02%	0.62%	0.39%	-3.40%	0.41%	4.00%	-0.02%	-2.99%
Repo Spread 2007 Q1Q2	2.79%	-3.50%	1.77%	-0.01%	-0.01%	-0.01%	-0.01%	-10.42%	0.00%	0.79%
Repo Spread 2007 Q3Q4	2.83%	-3.79%	1.74%	-0.13%	-0.28%	-0.25%	-0.27%	-3.79%	-0.51%	1.39%
Repo Spread 2008	2.84%	-3.60%	1.78%	-0.03%	-0.06%	0.54%	-0.12%	-0.41%	-0.07%	0.07%

Table 5: Percentage Changes of Target Moments When Benchmark Parameters are Perturbed -1%

Moments\Parameters	ρ^l	ν_1	ν_2	ν_3	ν_4	η	ϵ	φ	σ	σ_0
Mean Hair Cut 2007	-0.0242%	-0.0242%	-0.0242%	-0.0196%	-0.0196%	-0.0555%	-0.0108%	13.0235%	0.0495%	0.5497%
Mean Hair Cut 2008 Q1	-1.4104%	-1.4105%	-1.4105%	-1.3913%	-1.3926%	-1.9557%	-1.2713%	-3.4882%	-1.0761%	-1.3745%
Mean Hair Cut 2008 Q2	-4.7988%	-4.7976%	-4.7980%	-4.8009%	-4.8012%	-8.8158%	-4.3371%	-3.0797%	-4.5819%	-5.2090%
Mean Hair Cut 2008 Q3Q4	-0.0676%	-0.0675%	-0.0675%	-0.0684%	-0.0681%	-0.6355%	-0.0406%	0.1936%	-0.0527%	-0.0768%
LIB-OIS 2007 Q1Q2	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
LIB-OIS 2007 Q3Q4	-0.1988%	-0.0553%	-0.0998%	-0.6825%	-0.4989%	-0.1835%	-0.0275%	-1.5768%	-0.0266%	-4.7096%
LIB-OIS 2008	4.1670%	4.2848%	4.2239%	3.6348%	3.8889%	7.1583%	3.8511%	-1.9889%	4.3769%	6.0964%
Repo Spread 2007 Q1Q2	-21.3727%	3.6159%	-1.7577%	0.0149%	0.0149%	0.0153%	0.0162%	10.3965%	-0.0294%	0.0013%
Repo Spread 2007 Q3Q4	-18.3437%	3.3505%	-2.2964%	-0.5010%	-0.4287%	-0.4130%	-0.4179%	3.1160%	-0.1401%	-1.8756%
Repo Spread 2008	-6.2620%	3.0546%	-2.4218%	-0.6207%	-0.5981%	-1.0617%	-0.5677%	-0.5444%	-0.5526%	-0.8261%

6 Computation and Simulation

6.1 The Simulation Method

I delegate the detailed description of the computation method to appendix D.1. Given a numerically solved general equilibrium, I describe the simulation method. The time $t = 0$ is the first week of January 2007 and the simulation ends at the third week of November 2008, a total of 99 periods. Given an equilibrium, a simulated path is fixed by two things. One is the exogenous shock path containing μ_t and $\delta_{t,1}$ for all t and the other is the initial endogenous state $(B_0, K_{0,1}, K_{0,2})$. As mentioned in the section 3.3, I use a realization that $\mu_t \equiv 0$ for the aggregate component of random utilities. $\delta_{t,1}$ path is obtained from approximating the true ABX time series by the grids $\delta_{t,1}(k)$ for $k = 1, ..., 10$ defined in the previous section.

Instead of taking the common approach that using the steady-state[20] as candidate for endogenous initial state, I did something different. The reason is the following. Since I assume agents have rational expectations on the evolution of $s_{t+1}|s_t$, they are aware with the Markov process $\delta_{t,1}$ and its transition matrix. In any steady state, however, agents believe that $\delta_{t,1}$ is a constant. If I use the steady-state as the initial state, at time 1, I should observe an immediate change in all equilibrium objects even if the realization of $\delta_{t,1} = \delta_{t,0}$. This is generated from the arrival of information about the $\delta_{t,1}$ process. That large initial change is not observed in the data.

To get $(B_0, K_{0,1}, K_{0,2})$, I use the endogenous state obtained from simulating the equilibrium path with long enough $\delta_{t,1} = \bar{\delta}_1$ realizations. Numerical tests suggest that the state I obtained from the above procedure is not sensitive to where I start from. Though it is not the goal of this paper, the model is able to generate an acute rise in both the repo issuance and the RMBS stocks if I start from a low level of repo liability and asset holdings. That is, from the perspective of this model, the structural banking boom before 2007 is consistent with rational expectations and is not necessarily associated with asset bubbles.

Policy functions and value functions off the grids are calculated by linear interpolations. I check errors of such interpolations and find they are very small. Expectations with respect to the Gaussian distributions are computed with Gauss–Hermite quadrature with 15 grid points.

[20] A steady-state is the equilibrium that $\delta_{t,1} \equiv \delta_1$ for some constant δ_1.

6.2 Results

In this subsection, I report the simulation result from the calibrated benchmark model and compare it with available data observations. First of all, the table 6 compares the simulated moments and their targets. Considering my model is parsimonious, it did reasonably well in fitness. Figure 4 illustrates the fitness of the other two targeted paths: the shock path $\delta_{t,1}$ and the average haircut path H_t.

Table 6: Simulated Moments VS Targets

	Targets			Simulation		
	Jan-June 2007	Jul-Dec 2007	2008	Jan-June 2007	Jul-Dec 2007	2008
Repo Rate	6.41bp	76.35bp	199.44bp	16.62bp	45.69bp	205.63bp
LIB-OIS	7.97bp	58.71bp	108.1bp	0bp	50.91bp	111.65bp

From the figure, the model misses a significant part of the haircut jump in the September of 2008. It may come from several reasons. First of all, notice that the calibrated shock path $\delta_{t,1}$ after the September 2008 doesn't change a lot. Since the $\delta_{t,1}$ shock is the only aggregate shock in my model, it is not surprising that endogenous variables do not respond too much after that date. Secondly, the timing of the large haircut jump matches perfectly with events that may trigger panic on the market. On September 20th, Lehman Brothers officially filed the bankruptcy. Notice that my model only contains the fundamental-based run and has unique equilibrium on the repo market by assumption 1. The under-performance of the model after the September 2008 may suggest the importance of the panic-based runs.

Figure 5 to 7 depict other important simulated equilibrium paths that are not targeted in the calibration. Though these values are not targeted, they are consistent with many pieces of evidence in the literature. For example, Gorton et al. (2020) documented that, at the end of 2008, the total repo liability dropped around 40% compared with its pre-crisis level. The trend of the average transaction price of RMBS by insurance companies reported in Merrill et al. (2013) is very similar to my simulated path. Another sanity check is to compare the pre-crisis level of the total issuance of the RMBS (1266.8 billions) with the total issuance in 2007 and 2008 in Ospina and Uhlig (2018). My simulated path produced that the relative total issuance of the RMBS in 2007 and 2008 are $\frac{897.09}{1266.8} = 70.1\%$ and $\frac{158.855}{1266.8} = 12.5\%$, roughly in line with their data counterparts 75.8% and 8.4%.

7 Counter-factual Experiments

In this section, I conduct four counterfactual experiments to decompose the price, the liquidity, and the run factors. In the first experiment, I solve the equilibrium without asymmetric information friction on the asset market. The equilibrium concept and the solution method are similar with the benchmark model. Specifically, the partial equilibrium on the asset market implies

$$p_j^{NAI}(s_t) = \frac{\rho^h \mathbb{E}_t[v_1^{b,NAI}(s_{t+1})]}{\lambda^{NAI}(s_t)}, \text{ for all } j \in \{1,2\} \tag{38}$$

$$\theta_1^{NAI}(s_t) = \theta_2^{NAI}(s_t) = \frac{Cash^{NAI}(s_t)}{\sum_j p_j^{NAI}(s_t) M_j}. \tag{39}$$

That is, the liquidity for both assets is the same and their quality differences are completely compensated by the price gap. Notice that the liquidity of both assets can still fluctuate in the equilibrium due to the aggregate demand conditions. For the simulation, to be comparable, I start from the same initial state. Instead of only feeding in $\delta_{t,1}$ path as what I do in the benchmark model, I also injected the $K_{t,1}, K_{t,2}$ path obtained in the benchmark simulation. The path of B_t is still endogenous. It isolates the long-run effect of liquidities through asset accumulation. Otherwise, I would see an increase in the asset stock during the crisis because the initial state of the simulation is not correctly specified.

The second counterfactual experiment uncovers the contribution of the run factor induced by the strategic complementarity among lenders, which is controlled by the parameter φ. I solve the equilibrium with $\varphi = 0$ and simulate it in the same way with the benchmark model. The third experiment simply combines the first two. The gap between the result from this experiment and the benchmark model contains the price factor and the general equilibrium effect between two markets due to the endogenous balance sheet dynamics. To further isolate the price factor, the fourth experiment assumes away the stochastic movement of $\delta_{t,1}$. That is, I calculate the simulated path under the following conditions: (1) the shock path $\delta_{t,1} \equiv \delta_{0,1}$ for all t; (2) B_t is endogenously determined but I use the $K_{t,1}$ and $K_{t,2}$ generated from the benchmark model simulation; and (3) asset types are common knowledge

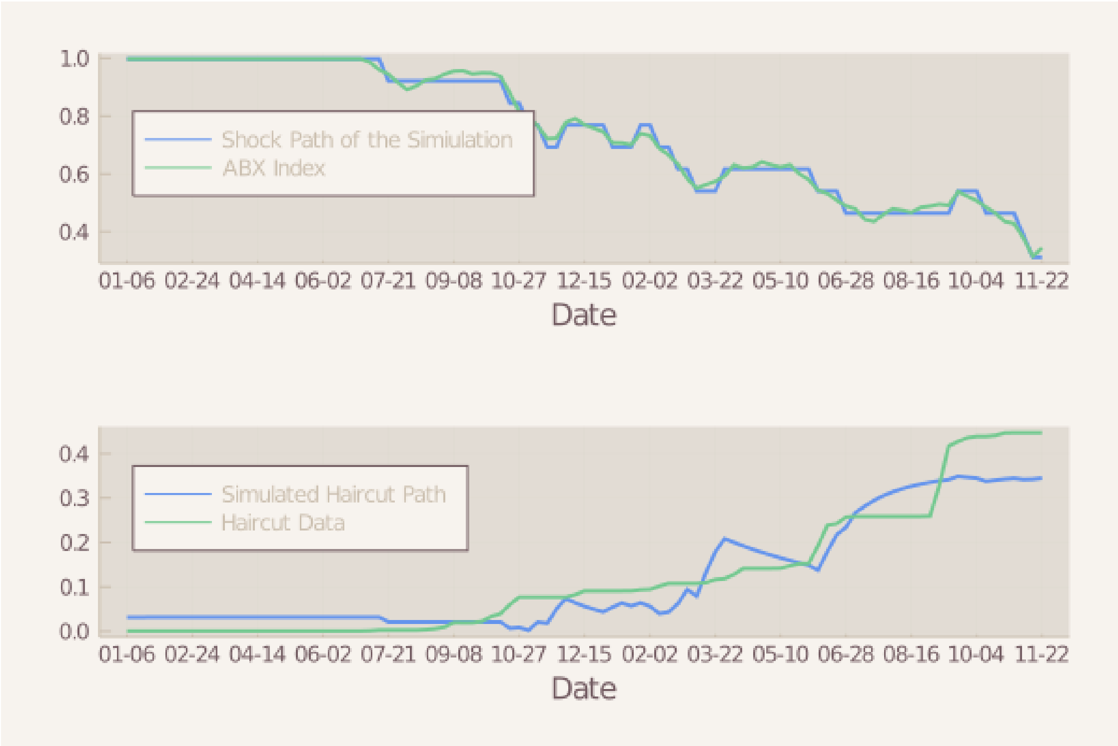

Figure 4: Fitness of Targeted Variables

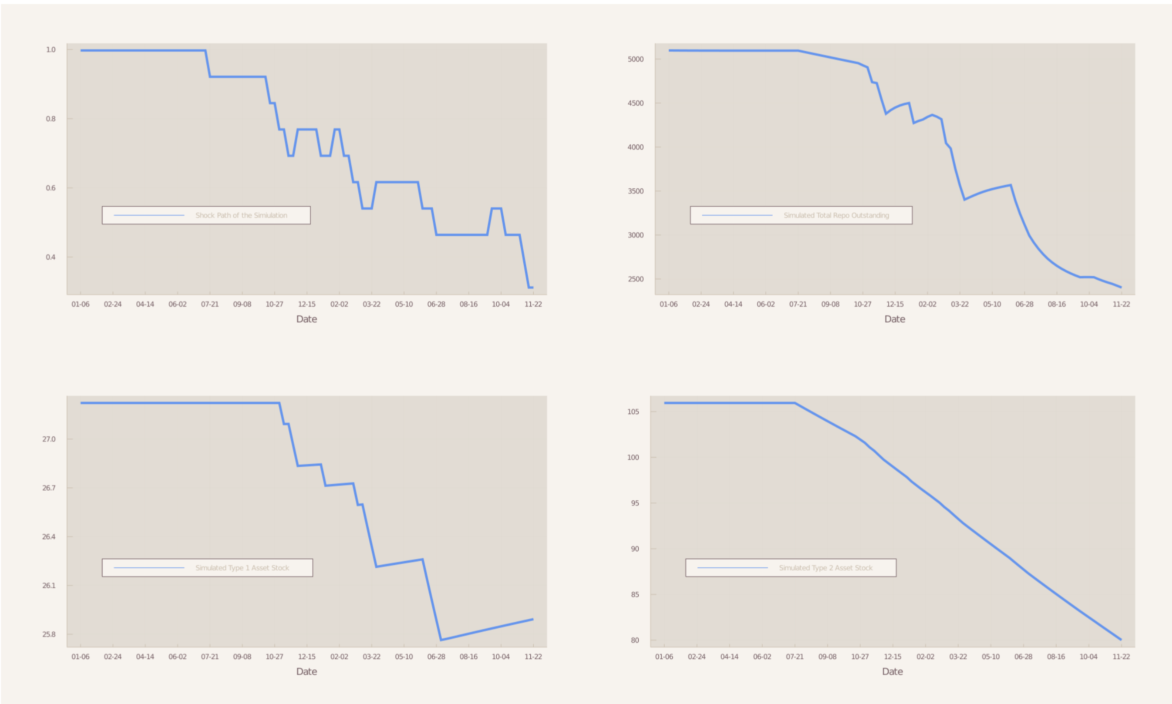

Figure 5: Simulated Path: State Variables s_t

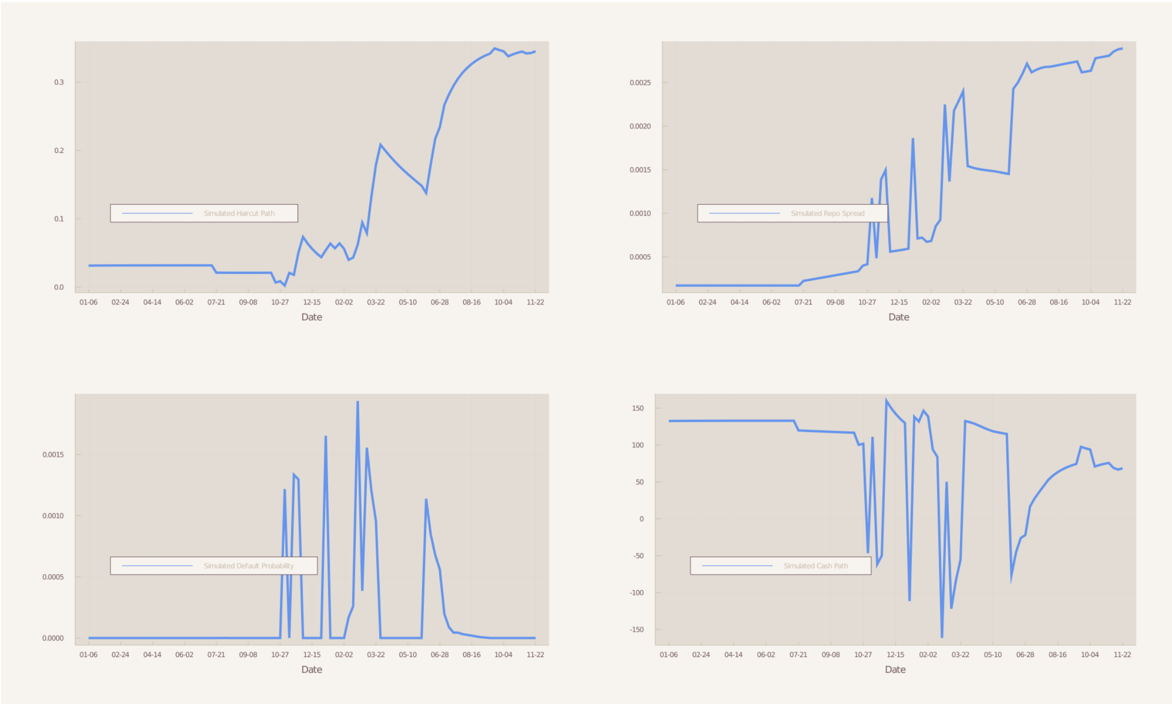

Figure 6: Simulated Path: Repo Market Variables

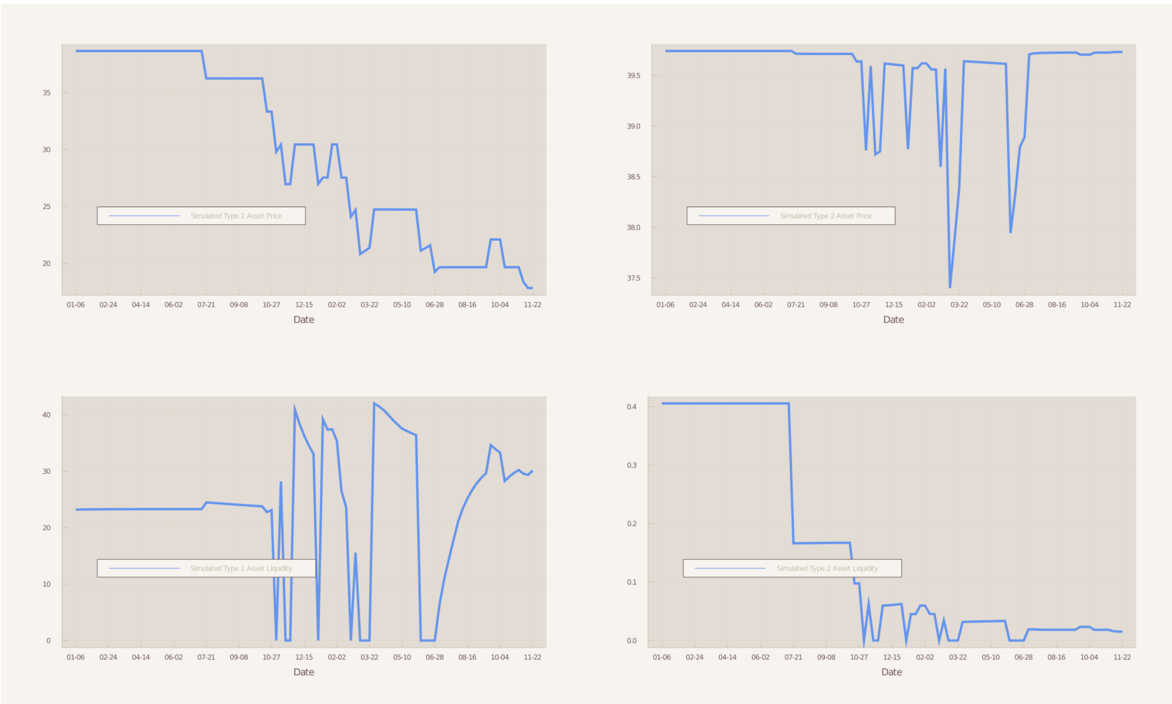

Figure 7: Simulated Path: Asset Market Variables

and $\varphi = 0$.

The decomposition goes in the following way. Consider an interested variable X. Suppose the simulated path of X in the benchmark simulation is X_t and its data observation is X_t^{Data}. Let X_t^{NAI} be the simulated path obtained from the first experiment described above. I denote X_t^{NSC} as the simulated path generated from the second experiment. Let $X_t^{P\&G}$ be the simulated path of the third experiment. I assume the simulated path generated from the fourth experiment is X_t^{GE}. I obtain

$$\begin{aligned}
\text{Liquidity factor contribution} &= X_t - X_t^{NAI}; \\
\text{Run factor contribution} &= X_t - X_t^{NSC}; \\
\text{Price factor contribution} &= X^{P\&G} - X_t^{GE}; \\
\text{General Equilibrium factor contribution} &= X_t^{GE}; \\
\text{Residual} &= X_t^{Data} - X_t.
\end{aligned}$$

I consider the decomposition of three factors for the haircut path h_t, the total repo liability B_t, the repo rate R_t^*, and the default probability path π_t. Figure 8 to 11 summarize the result.

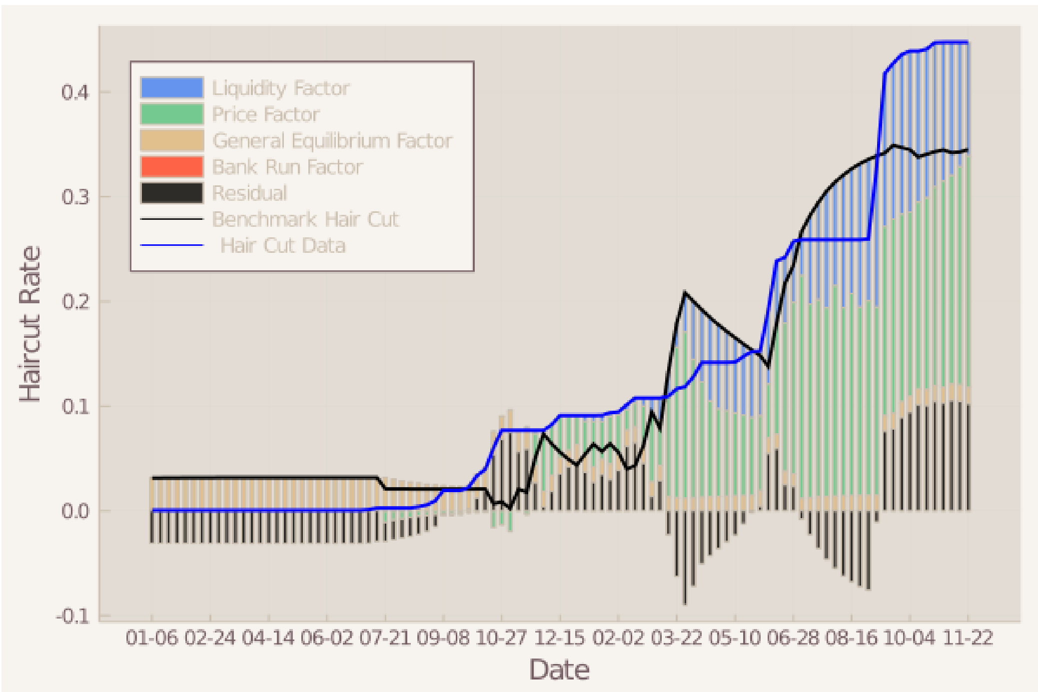

Figure 8: Decomposition of Haircut Rate to Three factors

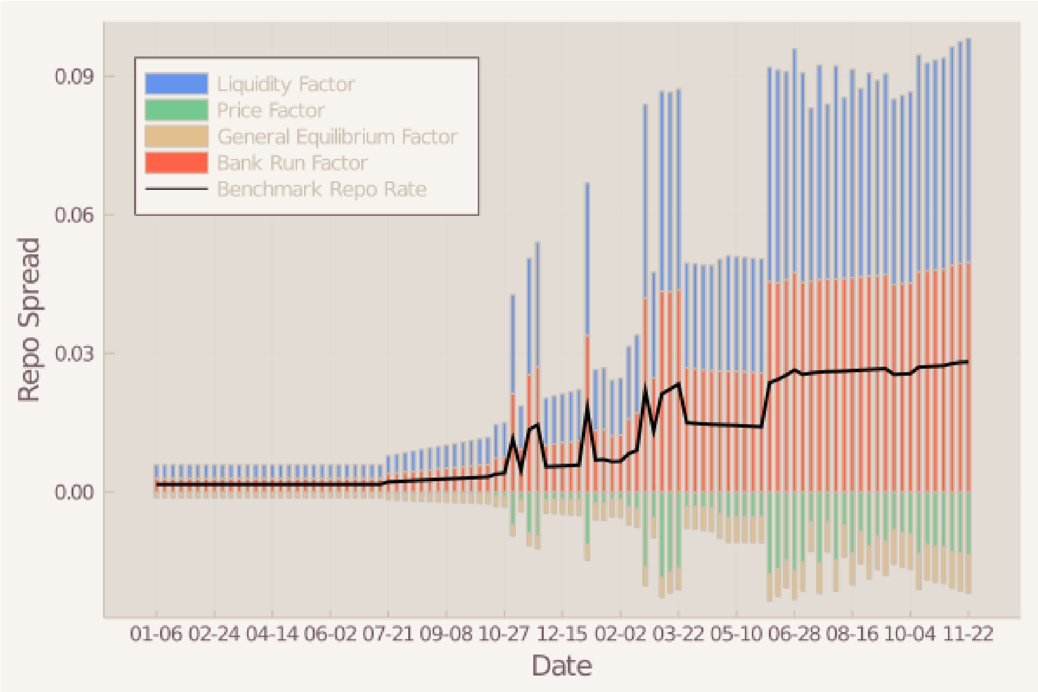

Figure 9: Decomposition of Repo Rate to Three factors

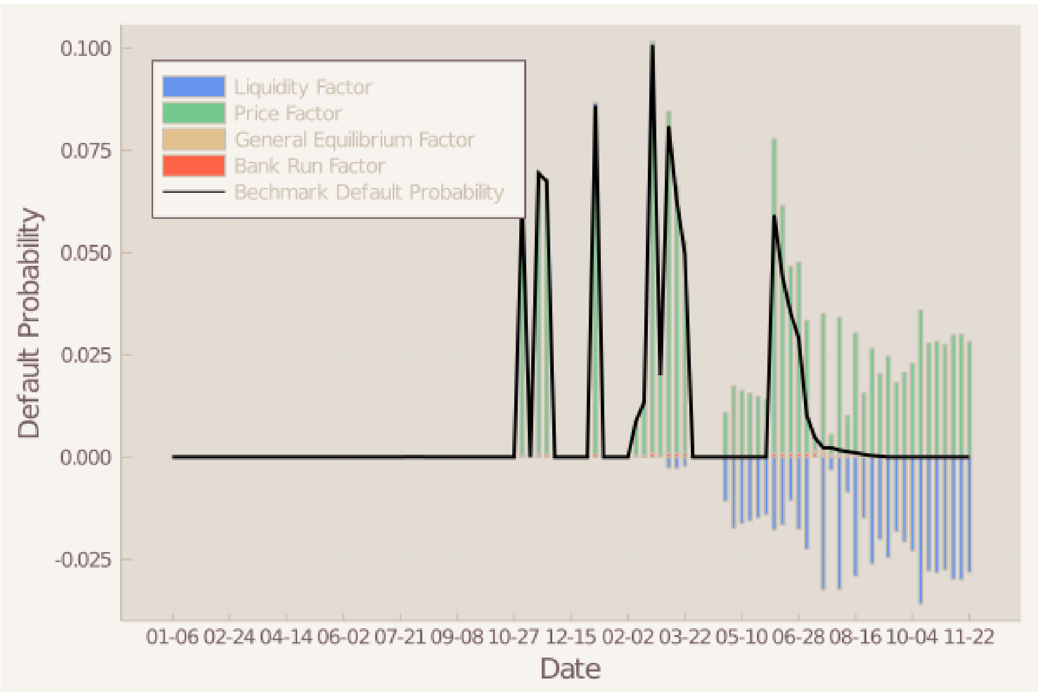

Figure 10: Decomposition of Default Probability Rate to Three factors

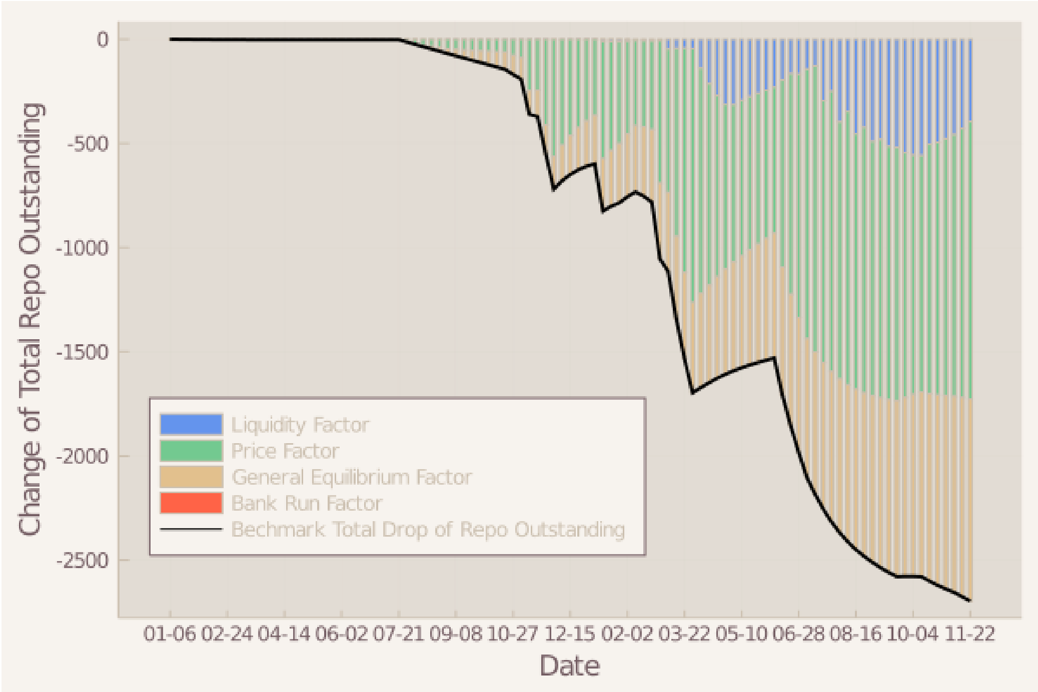

Figure 11: Decomposition of Repo Outstanding Drop to Three factors

The following results deserve special attention. Firstly, the liquidity drying up caused by asymmetric information plays a crucial role in every aspect of the repo market crash. It explains 30%[21] of the increase in haircut, 13% of the drop in total repo outstanding, and a large part of the increase in repo spread. Secondly, throughout the crisis, the fundamental-based run has a significant and persistent effect on the repo rate but only a small effect on the repo haircut. Thirdly, the general equilibrium effect generated from the interactions between the RMBS market and the repo market explains 33% of the drop in total repo outstanding. This result, in hindsight, confirms that it is important to study the RMBS market and the repo market together rather than separately. Fourthly, the price factor is the main reason for the default probability surge in the early stage of the crisis.

It is useful to discuss why does the second result emerge, I revisit the buyer's optimal decision on the repo market in problem 1. It is obvious that buyers utilize all their available assets as collateral. So the first constraint in problem 1 is binding. The main trade-off for a buyer is between two contracts: one with a higher repo spread, less collateral portfolio per unit repo contract, and a larger issuance volume, and another with lower repo spread, more col-

[21]Measured by the ratio between the blue area and the total area. The following numbers are calculated similarly.

lateral portfolio per unit, and hence smaller issuance volume. Collateral portfolio per unit of repo contract is bounded downward by $C(s_t, (k_j^*)_j) \geq 1$. I notice that this constraint is always binding on the simulated path. That is, calibrated parameters suggest buyers always favor the first contract over the second one in the above trade-off. Buyers cannot issue more repo contracts not because they optimally choose to do so in exchange for a lower coupon rate, but they are constrained by the accounting rules that assets have to be marked to market. These accounting rules have nothing to do with the strategic complementarity among lenders. Therefore it is natural that shutting down φ does not influence the extensive margin of a repo contract, i.e. the haircut rate and total issuance. This intuition is supported by literature on the fair value accounting rule during crisis time such as Kolasinski (2011) and many papers referenced therein.

8 Policy Implications

Policy implications are straightforward. Bailing out the troubled banks during the great recession is not as costly as previously believed. My results support various liquidity programs initiated by the FED during the crisis and predict the effectiveness of those interventions in mitigating the increase of haircuts. However, the second result in the previous section suggests that conventional monetary policies are ineffective in easing the haircut surge. Federal fund rate cuts increase the repo spread, which has little effect on the haircuts. Notice that the ineffectiveness is independent of zero lower bounds.

My model is also able to analyze macro-prudential policies related to the repo market. Suppose buyers carry exogenous cash reserve $CR(s_t)$ when state is s_t. And assume cash reserve cannot be consumed or used for investment. They are only available when buyers' new repo issuance and assets dividends are not enough to cover the maturing repo liabilities. Let us denote the new aggregate state as $\bar{s}_t = (s_t, CR(s_t))$. The equilibrium definition is an immediate generalization from the definition 3 with one modification[22].

$$Cash(\bar{s}_t, NR^*) = \begin{cases} Cash(s_t, NR^*) & \text{if } Cash(s_t, NR^*) > 0 \\ \min\{CR(s_t) + Cash(s_t, NR^*), 0\} & \text{otherwise} \end{cases}.$$

Let us denote a buyer family's total issuance of repo contract after deducting

[22]Notice that if $CR(s_t) \equiv 0$, it coincides with definition 3.

coupon payments as $TR^*(s_t)$. I have

$$TR^*(s_t) = \left(1 - \frac{R^*(s_t)}{1-\rho^h(1-\beta)}\right) \cdot (1 - \pi(s_t, NR^*(s_t))) \cdot NR^*(s_t).$$

The following proposition uncovers the relationship of equilibrium between the model with cash reserve and the benchmark model.

Assumption 2. *$TR^*(s_t)$ is continuous and decreasing in B_t for all s_t.*

Proposition 4. *Take a benchmark equilibrium in definition 3. Consider any element of the benchmark equilibrium $X(s_t)$. Suppose the assumption 2 holds. There exists functions $B^*(s_t)$ and $B^{**}(s_t)$ such that $\bar{X}(\bar{s}_t)$ is the corresponding element of an equilibrium for the economy with cash reserve $CR(s_t)$, where*

$$\bar{X}(\bar{s}_t) = \begin{cases} X(s_t) & \text{if} \quad B_t \in [0, B^*(s_t)) \\ X(s_t^*) & \text{if} \quad B_t \in [B^*(s_t), B^{**}(s_t)) \\ X(s_t^{**}) & \text{if} \quad B_t \in [B^{**}(s_t), +\infty) \end{cases},$$

and

$$s_t^* = \left(B^*(s_t), (K_{t,j})_j, \delta_{t,1}\right)$$

$$s_t^{**} = \left(B_t - \frac{1}{\beta} CR(s_t), (K_{t,j})_j, \delta_{t,1}\right).$$

Functions $B^*(s_t)$ and $B^{**}(s_t)$ are uniquely determined by

$$TR^*\left(B^*(s_t), (K_{t,j})_j, \delta_{t,1}\right) + (1-\omega) \cdot \sum_j K_{t,j}\delta_{t,j} = \beta \cdot B^*(s_t)$$

and

$$B^{**}(s_t) = B^*(s_t) + \frac{CR(s_t)}{\beta}.$$

The proof is delegated to the appendix E. Direct computation of the equilibrium with cash reserve might be difficult since the dimension of the state space in the benchmark model is already high. Therefore proposition 4 is convenient. I can analytically construct the equilibrium with cash reserve from the benchmark equilibrium if an easy-to-verify condition is satisfied. The intuition is the following. Notice that $CR(s_t)$ only enters the equilibrium indirectly through the term *Cash*. So if $\bar{s}_t$ and s_t imply the same *Cash* for buyers, the two equilibrium are observational equivalent.

9 Discussion

In the decomposition exercise, the contribution of each factor is calculated assuming the existence of policy interventions. It is also interesting to ask what will happen if a certain policy is not implemented during the crisis. So it is useful to discuss the robustness of the calibration method to different policy responses. All calibration targets and simulations are within the time window of January 2007 to December 2008. So I confine my discussion on interventions implemented before 2009. US policymakers intervened in four major ways in this time: fed funds interest rate cuts; liquidity programs to keep the financial institution operating; guarantee programs to support the critical funding markets for financial institutions; and conservatorship of systematic institutions like Fannie and Freddie[23].

I consider these policies one by one. An unexpected fed fund rate cut translates to a one-time jump of $R^*(s_t)$ in my model, which represents the spread between the repo rate and the lender's outside option. Since I target the average $R^*(s_t)$ for at least six months, the impact of such intervention should be small. Among all liquidity programs[24], TSLF and TALF are directly related to the assets considered in my model (private-labeled RMBS). Though announced in November 2008, TALF commenced operation in March 2009. So it does not bias the calibration too much. TSLF is only open to primary dealers, I expect it has a similar effect with PDCF. Both of these facilities alleviate the pain of raising funds outside the repo market for buyers, pushing up the value of η in my model. Guarantee programs related to Bear Stearns is one example of how buyers fulfill their repo obligations when $Cash < 0$. To fix such bias, I can introduce permanent shocks to η when policies are implemented. Since the whole path of LIB-OIS spread is observable, these shocks are easily identified with a similar calibration method in section 5.3. Other liquidity programs, guarantee programs, and conservatorship of systematic institutions do not have a direct influence on my calibration strategy.

[23] See the programs archive summarized by the Federal Reserve Bank of New York.

[24] These include CPFF (Commercial Paper Funding Facility), LSAP (Large Scale Asset Purchase), MMIFF (Money Market Investor Funding Facility), PDCF (Primary Dealer Credit Facility), TALF (Term Asset-Backed Securities Loan Facility), and TSLF (Term Security Lending Facility).

10 Concluding Remarks

Three factors are commonly believed to be important for the repo market crash: price, liquidity, and run. In this paper, I constructed a parsimonious model which integrated the RMBS market with asymmetric information and the repo market with strategically complementary lenders. It nested these three factors under a unified framework and tracked their interactions. I proposed a simple concept of equilibrium and characterized its properties.

Beyond theoretically tracking the interactions of the three factors using the structural model, I also quantitatively evaluate their contributions. The model yielded three crucial results. First, besides the contribution of the price factor, the liquidity drying up caused by asymmetric information plays a crucial role in every aspect of the repo market crash. It explains 30% of the increase in haircut, 13% of the drop in total repo outstanding, and a large part of the increase in repo spread. Second, throughout the crisis, the fundamental-based run significantly affects the repo rate but only has a small effect on the repo haircut. Third, in addition to the three factors, the general equilibrium effect generated from the interactions between the RMBS market and the repo market explains 33% of the drop in total repo outstanding. I discussed the policy implications of these findings.

Bibliography

Bernanke, B. and M. Gertler (1989). Agency costs, collateral, and business cycle fluctuations. *American Economic Review 79*, 14–31.

Bernanke, B. and M. Gertler (1990). Financial fragility and economic performance. *The quarterly journal of economics 105*(1), 87–114.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic perspectives 23*(1), 77–100.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *The review of financial studies 22*(6), 2201–2238.

Chang, B. (2018). Adverse selection and liquidity distortion. *The Review of Economic Studies 85*(1), 275–306.

Cho, I.-K. and D. M. Kreps (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics 102*(2), 179–221.

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017). The great escape? a quantitative evaluation of the fed's liquidity facilities. *American Economic Review 107*(3), 824–57.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy 91*(3), 401–419.

Goldstein, I. and A. Pauzner (2005). Demand–deposit contracts and the probability of bank runs. *the Journal of Finance 60*(3), 1293–1327.

Gorton, G., T. Laarits, and A. Metrick (2020). The run on repo and the feds response. *Journal of Financial Stability 48*, 100744.

Gorton, G. and A. Metrick (2012). Securitized banking and the run on repo. *Journal of Financial economics 104*(3), 425–451.

Gorton, G. B. (2009). Slapped in the face by the invisible hand: Banking and the panic of 2007. *Available at SSRN 1401882*.

Guerrieri, V. and R. Shimer (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review 104*(7), 1875–1908.

Guerrieri, V. and R. Shimer (2018). Markets with multidimensional private information. *American Economic Journal: Microeconomics 10*(2), 250–74.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse selection in competitive search equilibrium. *Econometrica 78*(6), 1823–1862.

He, Z. and W. Xiong (2012). Dynamic debt runs. *The Review of Financial Studies 25*(6), 1799–1843.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review 102*(1), 238–71.

Keys, B., T. Mukherjee, A. Seru, and V. Vig (2008). Securitization and screening: Evidence from subprime mortgage backed securities. *Quarterly Journal of Economics 125*(1), 307–362.

Keys, B. J., T. Mukherjee, A. Seru, and V. Vig (2009). Financial regulation and securitization: Evidence from subprime loans. *Journal of Monetary Economics 56*(5), 700–720.

Keys, B. J., A. Seru, and V. Vig (2012). Lender screening and the role of securitization: evidence from prime and subprime mortgage markets. *The Review of Financial Studies 25*(7), 2071–2108.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of political economy 105*(2), 211–248.

Kolasinski, A. C. (2011). Mark-to-market regulatory accounting when securities markets are stressed: Lessons from the financial crisis of 2007–2009. *Journal of Accounting and Economics 52*(2-3), 174–177.

Li, Q. (2019). Securitization and liquidity creation in markets with adverse selection. *Available at SSRN 2932441*.

Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter (2016). Mortgage risk and the yield curve. *The Review of Financial Studies 29*(5), 1220–1253.

Martin, A., D. Skeie, and E.-L. v. Thadden (2014). Repo runs. *The Review of Financial Studies 27*(4), 957–989.

Merrill, C. B., T. Nadauld, R. M. Stulz, and S. M. Sherlund (2013). Why were there fire sales of mortgage-backed securities by financial institutions during the financial crisis. *Fisher College of Business Working paper*, 03–02.

Morris, S. and H. S. Shin (2001). Global games: Theory and applications.

Ospina, J. and H. Uhlig (2018). Mortgage-backed securities and the financial crisis of 2008: a post mortem. Technical report, National Bureau of Economic Research.

Saleuddin, R. and W. Jansson (2021). Revisiting subprime pricing irrationality during the global financial crisis. *The Journal of Financial Crises 3*(2), 1–40.

Shi, S. (2015). Liquidity, assets and business cycles. *Journal of Monetary Economics 70*, 116–132.

Stanton, R. and N. Wallace (2011). The bear's lair: Index credit default swaps and the subprime mortgage crisis. *The Review of Financial Studies 24*(10), 3250–3280.

Wang, Y. (2019). *Counterparty Risk and Repo Runs*. Ph. D. thesis, The University of Chicago.

Wiggins, R., T. Piontek, and A. Metrick (2014). The lehman brothers bankruptcy a: overview. *Yale program on financial stability case study*.

Williams, B. et al. (2016). Search, liquidity, and retention: Signaling multidimensional private information. In *2016 Meeting Papers*, Number 1413. Society for Economic Dynamics.

Wright, R., P. Kircher, B. Julien, and V. Guerrieri (2019). Directed search and competitive search: A guided tour. *Journal of Economic Literature*.

Appendices

A A Brief Review of the Shadow Banking System

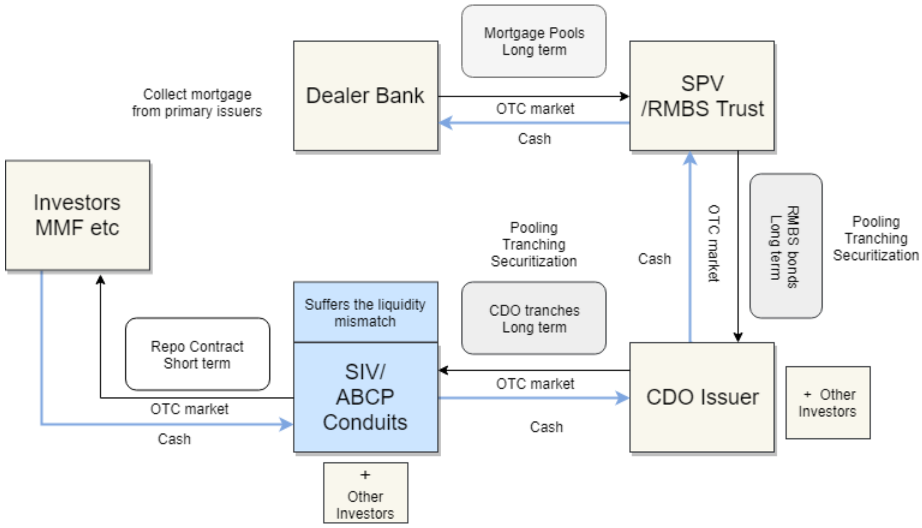

Figure 12: A Simplified Model of Shadow Banking

Figure 12 shows a simplified model of ”shadow banking” in the issuance of residential mortgages. **In the first step**, mortgages (either prime or subprime) are issued by local banks or mortgage companies. Different from ”traditional banking”, these local institutions no longer hold mortgages on balance sheets until their maturities. Instead, they package mortgages; sell these pools to dealer banks, and use proceedings to finance new issuance. **The second step** is conducted by dealer banks. They collect thousands of mortgages from primary originators in different locations all over the country. The idea is that this pooling process can eliminate or at least significantly reduce the idiosyncratic shocks of local housing markets. Then they transfer these pools to off-balance sheet vehicles, such as SPVs (special purpose vehicles) and RMBS (residential mortgage-backed security) trust. SPVs and RMBS trust are not operating entities in the usual context. They are robot companies (without employees or physical capital) with a set of pre-specified rules to fulfill a special purpose, typically to finish the securitization process and they are ”bankruptcy free”(not legally applicable to bankruptcy laws). **In the next step**, off-balance sheet vehicles slice the entire expected cash flow from mortgage pools into different debt securities with a hierarchy on the seniority. That is, senior debt holders will be paid before the subordinate debt holders. As argued by Li (2019), this tranching and securitization practice mitigates the adverse selection and improves the allocative efficiency in the market. The cash received from selling RMBS bonds is transferred back to the dealer bank, which is used to buy mortgage pools in the future. **The fourth step** is similar to the third step. It is another round of pooling, tranching, and securitization. SPVs sell RMBS tranches to CDO (Collateralized Debt Obligation) issuers. CDO issuers then combine these RMBS tranches with other assets like credit card receivables, student loans, and auto loans, etc into a large cash pool. Still, pooling is aimed to hedge the risk from housing markets and other markets. They slice and issue CDO tranches backed by this cash pool. Proceedings are used to buy RMBS tranches for issuing future CDOs. **The last link of the chain** is implemented by another group of off-balance sheet vehicles: the SIVs (special investment vehicles), ABCP (asset-backed commercial paper) conduits, CDO put providers and SIV Lites. I will not distinguish these different institutions in this paper, but rather focus on the similar role they have played in the shadow banking system. They are similar in the following sense. (1) They are actively managed by a manager (therefore different from SPVs). (2) They invest in a portfolio of long-term assets such as the RMBS tranches and CDO tranches. (3) There exists a set of pre-defined restrictions on the portfolio. Typically restrictions are on maturity, diversification, risk exposure, the leverage rate and etc. (4) They are typically highly leveraged. (5) They finance the purchase of portfolio by issuing short-term debt – the repo agreement – to lenders like MMF (money market funds) on the repo market. The repo agreement is a form of collateralized short-term (the maturity varies from one day to a year) borrowing contract. The seller of a repo contract borrows cash from the repo buyer, while at the same time he transfers some assets to repo buyers as collateral. At maturity, if the repo seller defaults, the repo buyer can keep the asset; otherwise, the ownership of the asset transfers back to the borrower. Therefore, these institutions suffer a liquidity mismatch. (6) They are bankruptcy-free. **In summary**, the SIVs are ″banks″ in this shadow banking system. ″Depositors″ save cash through repo contracts or asset-backed commercial papers. ″Loan″ is granted to originators of mortgage issuers via CDO tranches and RMBS tranches which are created by off-balance sheet vehicles.

B Partial Equilibrium on Asset Market

Let $\mathbb{S} \equiv \mathbb{R}_+ \times \mathbb{R}_+^J \times \{\underline{\delta}_1, ..., \bar{\delta}_1\}$ be the state space. Obviously $\mathbb{S}$ is a complete separable metric space. Denote its usual σ-Algebra as $\mathcal{S}$. Let $T(S|s_t) : \mathbb{S} \times \mathcal{S} \mapsto [0, 1]$ be the exogenously given transition function for aggregate states, where $S \in \mathcal{S}$. According to the Kolmogorov extension theorem, there exists a unique probability measure Pr on the measurable space $(\mathbb{S}^\infty, \mathcal{S}^\infty)$ such that $\Pr(S|s_t)$ coincide with $T(S|s_t)$ for all $S \in \mathcal{S}$ and $s_t \in \mathbb{S}$. With these notations, I state the corresponding sequence form of problem $\mathcal{P}_j$. To highlight the dependence relationship, I sometimes separate the state s_t into two parts: $\delta_{t,1}$ and $\lambda(s_t)$. Taking $\lambda(s_t)$ as given, $V_j^s(\delta_{t,1}, \lambda(s_t))$ is the solution to the following problem

$$\max_{\theta_j(s_{t+l}), p_j(s_{t+l})} \mathbb{E}\Big[\sum_{l=0}^{\infty}\big[\prod_{\tau=0}^{l}(1 - \min\{\theta_j(s_{t+l}), 1\})[\rho^l(1-\alpha)]^l \\ \cdot \{\delta_{t+l,j} + \min\{\theta_j(s_{t+l}), 1\} p_j(s_{t+l})\}\big]\Big] \tag{40}$$

with constraint

$$\lambda(s_{t+l}) \leq \frac{\min\{\theta_j^{-1}(s_{t+l}), 1\} \cdot \rho^h \mathbb{E}_{t+l}[v_j^b(s_{t+l+1})]}{p_j(s_{t+l})}, \tag{41}$$

and an incentive compatibility constraint that for all $j' < j$, type j' seller has no incentive to mimic the policy $\{\theta_j(\cdot), p_j(\cdot)\}$. Note that there is a gap between this constraint and the one stated in equation (26). Essentially, (26) only excludes the "one-shot deviations" by type j' sellers. However, thanks to the one shot deviation principle, constraint (26) is sufficient. I restate it here,

$$\frac{1}{1-\alpha} V_{j'}^s(\delta_{t+l,1}, \lambda(s_{t+l})) \geq \delta_{t+l,j'} + \min\{\theta_j(s_{t+l}), 1\} p_j(s_{t+l}) \\ + \rho^l(1 - \min\{\theta_j(s_{t+l}), 1\}) \mathbb{E}_{t+l}\big[V_{j'}^s(\delta_{t+l+1,1}, \lambda(s_{t+l+1}))\big]. \tag{42}$$

where $V_{j'}^s(\cdot)$ for $j' < j$ are defined recursively by mathematical induction. The expectation is well-defined in the probability space $(\mathbb{S}^\infty, \mathcal{S}^\infty, \Pr)$. For all s_t, let us define $\bar{\lambda}(s_t)$ as

$$\bar{\lambda}(s_t) \equiv \frac{\rho^h \mathbb{E}_t[v_1^b(\delta_{t+1,1})]}{(1-\alpha)\rho^l \mathbb{E}_t\left[\delta_{t+1,1} + \frac{\rho^h \mathbb{E}_{t+1}[v_1^b(s_{t+2})]}{\lambda(s_{t+1})}\right]};$$

and $\underline{\lambda}(s_t) \equiv \max\{1, \rho^h \mathbb{E}_t[\lambda(s_{t+1})]\}$.

Lemma 1. *Suppose $\{V_j^s(\cdot)\}_{j=1}^J$ are solutions to problem $\{\mathcal{P}_j\}_{j=1}^J$ as defined in problem 3. I have $V_1^s(\cdot) < V_2^s(\cdot) <, ..., < V_J^s(\cdot)$.*

Proof. First, from the standard equivalence relationship between the recursive form problem and the sequence form problem, I observe that problem $\mathcal{P}_j$ defined in problem 3 is equivalent to the problem (40). Therefore, it is sufficient to prove that, solutions to (40) with constraints (41) and (42) satisfy $V_1^s(\cdot) < V_2^s(\cdot) <, ..., < V_J^s(\cdot)$.

I proceed by mathematical induction. For $j = 1$, there is nothing to prove. Suppose that for $j' \leq j-1$, the condition is true. I show that $V_{j-1}^s(\cdot) < V_j^s(\cdot)$. Let $\{\theta_{j-1}(\cdot), p_{j-1}(\cdot)\}$ be the optimal policy chosen by type $j-1$ sellers. I verify that, this policy satisfies both the constraint (41) and (42).

For the constraint (41), note that $\{\theta_{j-1}(\cdot), p_{j-1}(\cdot)\}$ satisfies a similar constraint in problem $\mathcal{P}_{j-1}$, which suggests, for any state s_{t+l},

$$\begin{aligned}\lambda(s_{t+l}) &\leq \frac{\min\{\theta_{j-1}^{-1}(s_{t+l}), 1\} \cdot \rho^h \mathbb{E}_{t+l}[v_{j-1}^b(s_{t+l+1})]}{p_{j-1}(s_{t+l})} \\ &< \frac{\min\{\theta_{j-1}^{-1}(s_{t+l}), 1\} \cdot \rho^h \mathbb{E}_{t+l}[v_j^b(s_{t+l+1})]}{p_{j-1}(s_{t+l})}.\end{aligned}$$

The second inequality comes from the fact that

$$\mathbb{E}_{t+l}[v_{j-1}^b(s_{t+l+1})] < \mathbb{E}_{t+l}[v_j^b(s_{t+l+1})],$$

which is obvious from their definitions in equation (17).

For the constraint (42), note that from the (recursive-form) definition of $V_{j-1}^s(\cdot)$, I obtain

$$\begin{aligned}\frac{1}{1-\alpha} V_{j-1}^s(\delta_{t+l,1}, \lambda(s_{t+l})) &= \delta_{t+l,j-1} + \min\{\theta_{j-1}(s_{t+l}), 1\} p_{j-1}(s_{t+l}) \\ &+ \rho^l (1 - \min\{\theta_{j-1}(s_{t+l}), 1\}) \mathbb{E}_{t+l}\left[V_{j-1}^s(\delta_{t+l+1,1}, \lambda(s_{t+l+1}))\right].\end{aligned}$$

Hence, the incentive compatibility constraint between type $j-1$ and type j obviously holds. I move to IC constraints between type $j' < j-1$ sellers and type j sellers. Recall that $\{\theta_{j-1}(\cdot), p_{j-1}(\cdot)\}$ is optimal, thus naturally feasible in $\mathcal{P}_{j-1}$. IC constraints in $\mathcal{P}_{j-1}$ implies that, for all $j' < j-1$,

$$\frac{1}{1-\alpha} V^s_{j'}(\delta_{t+l,1}, \lambda(s_{t+l})) \geq \delta_{t+l,j'} + \min\{\theta_{j-1}(s_{t+l}), 1\} p_{j-1}(s_{t+l})$$
$$+ \rho^l (1 - \min\{\theta_{j-1}(s_{t+l}), 1\}) \mathbb{E}_{t+l}\big[V^s_{j'}(\delta_{t+l+1,1}, \lambda(s_{t+l+1}))\big],$$

which is what I want to prove. Thus it is feasible for type j sellers to take $\{\theta_{j-1}(\cdot), p_{j-1}(\cdot)\}$. This implies

$$V^s_j(\delta_{t+l,1}, \lambda(s_{t+l}))$$
$$\geq \mathbb{E}\Big[\sum_{l=0}^{\infty} \big[\prod_{\tau=0}^{l} (1 - \min\{\theta_{j-1}(s_{t+\tau}), 1\}) [\rho^l (1-\alpha)]^l$$
$$\cdot \{\delta_{t+l,j} + \min\{\theta_{j-1}(s_{t+l}), 1\} p_{j-1}(s_{t+l})\}\big]\Big]$$
$$> \mathbb{E}\Big[\sum_{l=0}^{\infty} \big[\prod_{\tau=0}^{l} (1 - \min\{\theta_{j-1}(s_{t+\tau})), 1\}) [\rho^l (1-\alpha)]^l$$
$$\cdot \{\delta_{t+l,j-1} + \min\{\theta_{j-1}(s_{t+l}), 1\} p_{j-1}(s_{t+l})\}\big]\Big]$$
$$= V^s_{j-1}(\delta_{t+l,1}, \lambda(s_{t+l})).$$

The first inequality holds since I have dropped the max operator in (40). The strict inequality comes from the condition $\delta_{t+l,j} > \delta_{t+l,j-1}$ for all state s_{t+l}. The third equality comes from the (sequence-form) definition of $V^s_{j-1}(\cdot)$. This completes the proof. □

Lemma 2. *For any state s_t, suppose $\underline{\lambda}(s_t) \leq \lambda(s_t) \leq \bar{\lambda}(s_t)$. I can establish the following upper bound for $V^s_1(s_t)$,*

$$V^s_1(s_t) \leq (1-\alpha)\{\delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v^b_1(s_{t+1})]}{\lambda(s_t)}\}.$$

Proof. Let $\theta_1(s_t)$ and $p_1(s_t)$ be the optimal policy function that achieves $V^s_1(s_t)$. First, I argue that, for all s_t,

$$p_1(s_t) \leq \frac{\rho^h \mathbb{E}_t[v^b_1(s_{t+1})]}{\lambda(s_t)}.$$

From the constraint (25) in problem $\mathcal{P}_1$, I obtain

$$
\begin{aligned}
p_1(s_t) &\leq \frac{\min\{\theta_1^{-1}(s_t), 1\}\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)} \\
&\leq \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)}
\end{aligned}
\tag{43}
$$

Since $V_1^s(s_t)$ is linear in $\theta_1(s_t)$, whenever $\theta_1(s_t) > 0$, I have

$$
V_1^s(s_t) = (1-\alpha)[\delta_{t,1} + p_1(s_t)] \leq (1-\alpha)[\delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)}],
$$

which is the result I want. The only case left is that sellers hold the asset forever, i.e $\mathbb{E}_t[\theta_1(s_{t+l})] = 0$ for all $l \geq 1$. However, from the sequence form definition of $V_1^s(s_t)$, I obtain

$$
\begin{aligned}
V_1^s(s_t) &= \mathbb{E}\Big[\sum_{l=0}^{\infty}[\rho^l(1-\alpha)]^l \delta_{t+l,1}\Big] \\
&= \lim_{L\to\infty}\Big\{\mathbb{E}\Big[\sum_{l=0}^{L}[\rho^l(1-\alpha)]^l \delta_{t+l,1}\Big] + \frac{[\rho^h]^L \mathbb{E}_{t+L}[v_1^b(s_{t+1+L})]}{\bar{\lambda}(s_{t+L+1})}\Big\} \\
&\leq (1-\alpha)[\delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\bar{\lambda}(s_t)}] \\
&\leq (1-\alpha)[\delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)}],
\end{aligned}
$$

where the inequality in the third line comes from repeatedly using $\lambda(s_t) \leq \bar{\lambda}(s_t)$ and iterating $\bar{\lambda}(s_t)$ forward. This completes the proof. □

Lemma 3. *For any state s_t and all j, suppose that $\lambda(s_t) < \bar{\lambda}(s_t)$, the constraint (25) is binding.*

Proof. I prove by contradiction. Consider the case $j = 1$. Let $\{\theta_1(\cdot), p_1(\cdot)\}$ be its optimal policy. Suppose that (25) is slack. Note that (25) is the only constraint in problem $\mathcal{P}_1$. I construct an alternative policy $\{\theta_1(\cdot), \tilde{p}_1(\cdot)\}$ such that it keeps the same probability of trade at all states; increases the selling price at state s_t so that (25) is binding; and at all other states $s_t' \neq s_t$, $\tilde{p}_1(s_t') = p_1(s_t)$. By construction, $p_1(s_t) < \tilde{p}_1(s_t)$. Substituting this back to (40), I discover that the value induced by the alternative policy is weakly better than the value from $\{\theta_1(\cdot), p_1(\cdot)\}$. To obtain the strict inequality for a contradiction, I verify that $\theta_1(s_t) > 0$. Since $V_1^s(s_t)$ is linear in $\theta_1(s_t)$, it is sufficient to compare two alternative plans at state s_t: $(0, \tilde{p}_1(s_t))$ and $(1, \tilde{p}_1(s_t))$. I have

$$
\begin{aligned}
&\delta_{t,1} + 1 \cdot \rho^l \mathbb{E}_t[V_1^s(s_{t+1})] \\
&\leq \delta_{t,1} + \rho^l (1-\alpha) \mathbb{E}\left[\delta_{t+1,1} + \frac{\rho^h \mathbb{E}_{t+1}[v_1^b(\delta_{t+2,1})]}{\lambda(s_{t+1})}\right] \\
&= \delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v_1^b(\delta_{t+1,1})]}{\bar{\lambda}(s_t)} \\
&< \delta_{t,1} + \frac{\rho^h \mathbb{E}_t[v_1^b(\delta_{t+1,1})]}{\lambda(s_t)} \\
&= \delta_{t,1} + \tilde{p}_1(s_t).
\end{aligned} \tag{44}
$$

Here the first line is the payoff (multiply by $\frac{1}{1-\alpha}$) for type 1 seller who adopts the policy $(0, \tilde{p}_1(s_t))$ at s_t. The inequality in the second line comes from the result in Lemma 2 for state s_{t+1}. The equality in the third line uses the definition of $\bar{\lambda}(s_t)$. The strict inequality in the fourth line of (44) is from the condition that $\lambda(s_{t+1}) < \bar{\lambda}(s_t)$. The last line is the seller's value of using policy $(1, \tilde{p}_1(s_t))$. This proves the the claim for case $j = 1$.

Now I focus on the case $j > 1$. Suppose that (25) is slack. Let $\{\theta_j(\cdot), p_j(\cdot)\}$ be its optimal policy. Consider an alternative policy function $\{\tilde{\theta}_j(\cdot), \tilde{p}_j(\cdot)\}$ which (i) coincides with the original optimal policy function on every state other than s_t; and (ii) has a slightly higher price but lower trading probability at s_t such that (41) holds with equality and

$$
\min\{\theta_j(s_t), 1\}\Big[p_j(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]\Big] \tag{45}
$$

$$
= \min\{\tilde{\theta}_j(s_t), 1\}\Big[\tilde{p}_j(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]\Big]. \tag{46}
$$

I first prove that $\{\tilde{\theta}_j(\cdot), \tilde{p}_j(\cdot)\}$ is feasible. By construction, type $j-1$ seller is indifferent between her own policy and $\{\tilde{\theta}_j(\cdot), \tilde{p}_j(\cdot)\}$. I only need to check that IC constraints hold for $j' < j-1$. Suppose for some j', (42) is violated

$$
\begin{aligned}
&V_{j'}^s(\delta_{t,1}, \lambda(s_t)) < \delta_{t,j'} + \min\{\tilde{\theta}_j(s_t), 1\}\tilde{p}_j(s_t) \\
&+ \rho^l (1 - \min\{\tilde{\theta}_j(s_t), 1\}) \mathbb{E}_t\big[V_{j'}^s(\delta_{t+1,1}, \lambda(s_{t+1}))\big].
\end{aligned}
$$

Since the IC constraint between type $j-1$ and type j holds for policy $\{\theta_j(\cdot), p_j(\cdot)\}$, I have

$$V^s_{j'}(\delta_{t,1}, \lambda(s_t)) \geq \delta_{t,j'} + \min\{\theta_j(s_t), 1\} p_j(s_t)$$
$$+ \rho^l (1 - \min\{\theta_j(s_t), 1\}) \mathbb{E}_t \big[V^s_{j'}(\delta_{t+1,1}, \lambda(s_{t+1})) \big].$$

Combining the above two inequalities, and substituting (45) in, I obtain

$$\big(\min\{\theta_j(s_t), 1\} - \min\{\tilde{\theta}_j(s_t), 1\}\big) \rho^l \mathbb{E}_t [V^s_{j'} - V^s_{j-1}] > 0.$$

However, as proved in the lemma 1, $\mathbb{E}_t[V^s_{j'} - V^s_{j-1}] < 0$ and by construction $\big(\min\{\theta_j(s_{t+l}), 1\} - \min\{\tilde{\theta}_j(s_{t+l}), 1\}\big) > 0$. This is a contradiction.

Hence, $\{\tilde{\theta}_j(\cdot), \tilde{p}_j(\cdot)\}$ is feasible. Again, from the single crossing condition implied by Lemma 1, I obtain

$$\min\{\theta_1(s_t), 1\} \Big[p_j(s_t) - \rho^l \mathbb{E}_t [V^s_j(s_{t+1})] \Big]$$
$$< \min\{\tilde{\theta}_1(s_t), 1\} \Big[\tilde{p}_j(s_t) - \rho^l \mathbb{E}_t [V^s_j(s_{t+1})] \Big],$$

which means type j seller is strictly better off with the alternative policy function. Thus I get a contradiction. This completes the proof. □

Lemma 4. *For any state s_t, suppose that $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$, the optimal policy satisfies (i) $\theta_1(s_t) = 1$, $\theta_j(s_t) \leq 1$ for all $j > 1$; and (ii) $p_j(s_t) < p_{j'}(s_t)$ for all $j < j'$.*

Proof. Let us focus on (i). I consider $\theta_1(s_t)$ first. Notice that there is only one constraint (25) and as proved by Lemma 3, it is binding. Substitute this constraint back to the objective function, I obtain

$$V^s_1(\delta_{t,1}, \lambda(s_t)) = \max_{\theta_1(s_t)} \Big\{ \delta_{t,1} + \min\{\theta_1(s_t), 1\} \Big[p_1(s_t) - \rho^l \mathbb{E}_t \big[V^s_1(\cdot) \big] \Big]$$
$$+ \rho^l \mathbb{E}_t \big[V^s_1(\delta_{t+1,1}, \lambda(s_{t+1})) \big] \Big\} (1 - \alpha), \tag{47}$$

where the price $p_1(s_t)$ is given by (43). I argue that the optimal policy $\theta_1(s_t) = 1$. Consider the optimization problem (47) in two separated intervals: $[0, 1]$

and $[1, \infty)$. If $\theta_1(s_t) \in [0, 1]$, the price $p_1(s_t)$ simplifies to

$$
\begin{aligned}
p_1(s_t) &= \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)} \\
&> \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\bar{\lambda}(s_t)} \\
&= \rho^l (1-\alpha) \mathbb{E}_t \left[\delta_{t+1,1} + \frac{\rho^h \mathbb{E}_{t+1}[v_1^b(s_{t+2})]}{\lambda(s_{t+1})} \right] \\
&\geq \rho^l \mathbb{E}_t[V_1^s(s_{t+1})],
\end{aligned}
$$

where the strict inequality uses the condition $\lambda(s_t) < \bar{\lambda}(s_t)$, and the inequality in the last line applies the result from Lemma 2. Since the objective function is linear in $\theta_1(s_t)$, the above inequality suggests $\theta_1(s_t) = 1$ is optimal within the range [0,1]. Suppose that $\theta_1(s_t) \geq 1$, the objective simplifies to $(1-\alpha)[\delta_{t,1} + p_1(s_t)]$. Recall that (43) is decreasing in $\theta_1(s_t)$ under the condition that

$$
\lambda(s_t) > \underline{\lambda}(s_t) \geq \rho^h \mathbb{E}_t[\lambda(s_{t+1})].
$$

That is, $p_1(s_t)$ is decreasing in $\theta_1(s_t)$. Thus, $\theta_1(s_t) = 1$ maximized the objective within $[1, \infty)$. Therefore, I have proved the optimal $\theta_1(s_t) = 1$.

Now I look at the case $j > 1$. I prove the desired result by contradiction. Denote the optimal policy at state s_t as $(\theta_j(s_t), p_j(s_t))$. Suppose that $\theta_j(s_t) > 1$. Combining with the binding constraint (41) proved in lemma 3, the objective function $V_j^s(\delta_{t,1}, \lambda(s_t))$ reduces to $(1-\alpha) \cdot [\delta_{t,j} + p_j(s_t)]$, where

$$
p_j(s_t) = \frac{\theta_j^{-1}(s_t) \cdot \rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{\lambda(s_t)},
$$

and the incentive compatibility constraint between type 1 seller and type j seller indicates

$$
V_1^s(\delta_{t,1}, \lambda(s_t)) = (1-\alpha)[\delta_{t,1} + p_1(s_t)] \geq (1-\alpha)[\delta_{t,1} + p_j(s_t)].
$$

Thus, I have $p_1(s_t) \geq p_j(s_t)$. From the proof in the first part that $\theta_1(s_t) = 1$, I get

$$\lambda(s_t) = \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{p_1(s_t)} < \frac{\rho^h \mathbb{E}_t[v_2^b(s_{t+1})]}{p_1(s_t)}$$
$$\leq \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{p_j(s_t)}. \tag{48}$$

This illustrates that the alternative policy $(1, p_j(s_t))$ is feasible and weakly better than the optimal policy $(\theta_j(s_t), p_j(s_t))$. However, (48) states that constraint (25) is slack for $(1, p_j(s_t))$, which is a contradiction to the Lemma 3. This completes the proof of the first part.

Let us prove (ii). For $j' > j$, according to Lemma 3 and the result in (i), I obtain

$$p_{j'}(s_t) = \frac{\rho^h \mathbb{E}_t[v_{j'}^b(s_{t+1})]}{\lambda(s_t)}$$
$$> \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{\lambda(s_t)} = p_j(s_t).$$

This completes the proof. $\square$

Lemma 5. *For any state s_t and all $j > 1$, suppose that $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$, the constraint (26) is binding between j and $j-1$ and slack otherwise.*

Proof. I separate the proof into two sub-cases: (i) (26) binds in problem $\mathcal{P}_2$, and (ii) for problem $\mathcal{P}_j$ with $j > 2$, (26) binds between j and $j-1$ and is slack otherwise. Consider the problem $\mathcal{P}_2$ first. For a contradiction, suppose that (26) is slack. Then I can prove $\theta_2(s_t) = 1$ by exact the same argument as the first part of the proof in Lemma 4. This implies

$$p_2(s_t) = \frac{\rho^h \mathbb{E}_t[v_2^b(s_{t+1})]}{\lambda(s_t)} > \frac{\rho^h \mathbb{E}_t[v_1^b(s_{t+1})]}{\lambda(s_t)} = p_1(s_t).$$

However, the objective function reads

$$V_1^s(s_t) = (1-\alpha)[\delta_{t,1} + p_1(s_t)] < (1-\alpha)[\delta_{t,1} + p_2(s_t)].$$

The incentive compatibility constraint is violated, which is a contradiction.

Let us consider the problem $\mathcal{P}_j$ for $j > 2$. I proceed by mathematical induction on j. Suppose the claim is true for all problem $\mathcal{P}_{j'}$ with $j' \leq j-1$. First, let us restrict to the situation where the IC constraint between type j and $j-2$ is binding. From the induction hypothesis, the IC constraint is also binding between type $j-1$ and type $j-2$ sellers, which implies

$$\begin{aligned}\frac{1}{1-\alpha}V^s_{j-2}(s_t) &= \delta_{t,j-2} + \theta_{j-1}(s_t)p_{j-1}(s_t) + \rho^l(1-\theta_{j-1}(s_t))\mathbb{E}_t\left[V^s_{j-2}(s_{t+1})\right] \\ &= \delta_{t,j-2} + \theta_j(s_t)p_j(s_t) + \rho^l(1-\theta_j(s_t))\mathbb{E}_t\left[V^s_{j-2}(s_{t+1})\right].\end{aligned}$$

From Lemma 4, $p_j(s_t) > p_{j-1}(s_t)$. I obtain $\theta_{j-1}(s_t) > \theta_j(s_t)$. According to the single crossing condition implied from Lemma 1, I get

$$\begin{aligned}\frac{1}{1-\alpha}V^s_{j-1}(s_t) &= \delta_{t,j-1} + \theta_{j-1}(s_t)p_{j-1}(s_t) + \rho^l(1-\theta_{j-1}(s_t))\mathbb{E}_t\left[V^s_{j-1}(s_{t+1})\right] \\ &< \delta_{t,j-1} + \theta_j(s_t)p_j(s_t) + \rho^l(1-\theta_j(s_t))\mathbb{E}_t\left[V^s_{j-1}(s_{t+1})\right],\end{aligned}$$

which is a contradiction since it violates the IC constraint between type $j-1$ and type j.

Next, suppose that the IC constraint between type j and $j-k$ for $2 < k \leq j-1$ is binding. I can replicate the above proof by arguing that the IC constraint between $j-k$ and $j-k+1$ is binding, which leads to a contradiction that the IC between $j-k+1$ and j is violated. Thus, the only case left is that the incentive compatibility constraints are slack between type j and all other types $j' < j$. The argument is exactly the same with that illustrated in the case for problem $\mathcal{P}_2$: I establish that $\theta_j(s_t) = 1$ and obtain a contradiction that the IC between type 1 and type j is violated. This completes the proof. $\square$

Lemma 6. *For any state s_t, assume that $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$. If the function $\lambda(s_t)$ is continuous in s_t and the transition function of aggregate state satisfies the Feller property, there exists a unique solution to problem $\mathcal{P}_j$ for all j. Moreover, functions $\left(V^s_j(s_t), p_j(s_t), \theta_j(s_t)\right)$ are all bounded and continuous in s_t.*

Proof. I prove by mathematical induction. Consider the case for $\mathcal{P}_1$. According to Lemma 1 to 5, I obtain $\theta_1(s_t) = 1$ and for all s_t,

$$p_1(s_t) = \frac{\rho^h \mathbb{E}_t[v^b_1(s_{t+1})]}{\lambda(s_t)}.$$

Substituting these back to the objective function, I obtain that $V_1^s(s_t) = (1 - \alpha)[\delta_{t,1} + p_1(s_t)]$, which is continuous and bounded.

Now suppose the claim is true for all $j' \leq j-1$, I prove the existence and uniqueness of solution to problem $\mathcal{P}_j$. Let $\left(V_{j'}^s(s_t), p_{j'}(s_t), \theta_{j'}(s_t)\right)$ be the solution to $\mathcal{P}_{j'}$. From the binding constraints (25) and (26), I can solve

$$p_j(s_t) = \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{\lambda(s_t)}, \tag{49}$$

$$\theta_j(s_t) = \theta_{j-1}(s_t) \cdot \left\{ \frac{p_{j-1}(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]}{p_j(s_t) - \rho^l \mathbb{E}_t[V_{j-1}^s(s_{t+1})]} \right\}. \tag{50}$$

Substituting these back to the objective function, the Bellman equation for $V_j^s(s_t)$ is

$$V_j^s(s_t) = \left\{ \delta_{t,j} + \theta_j(s_t) \Big[p_j(s_t) - \rho^l \mathbb{E}_t \big[V_j^s(s_{t+1}) \big] \Big] + \rho^l \mathbb{E}_t \big[V_j^s(s_{t+1}) \big] \right\} (1 - \alpha). \tag{51}$$

Denote $\mathcal{B}(\mathbb{S})$ as the space of all the bounded continuous functions mapping $\mathbb{S}$ to $\mathbb{R}$. Let d be a metric on $\mathcal{B}(\mathbb{S})$ generated by the sup norm. Thus $(\mathcal{B}(\mathbb{S}), d)$ is a complete metric space. Equation (51) maps an element of $\mathcal{B}(\mathbb{S})$ to itself. Obviously, the Bellman operator satisfies Blackwell's sufficient condition. Hence it is a contraction. According to the contraction mapping theorem, there exists a unique fixed point that defines $V_j^s(s_t)$.

This completes the proof. $\square$

Lemma 7. *For any state s_t, assume that $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$. If the function $\lambda(s_t)$ is continuous in s_t and the transition function of aggregate state satisfies the Feller property, there exists a partial equilibrium on the asset market as defined in definition 1.*

Proof. Let $\left(V_j^s(\delta_{t,1}, \lambda(s_t)), p_j(s_t), \theta_j(s_t)\right)$ be the solution to problem $\mathcal{P}_j$. I directly construct the partial equilibrium on the asset market. As suggested by the notation, sellers value functions $V_j^s(s_t)$ coincide with $V_j^s(\delta_{t,1}, \lambda(s_t))$. Define $\gamma_j(s_t, p) = 1$ if $p \in [p_j(s_t), p_{j+1}(s_t))$ for $j \in \{1, 2, ..., J-1\}$; $\gamma_1(s_t, p) = 1$ if $p \in [0, p_1(s_t))$; and $\gamma_J(s_t, p) = 1$ if $p \in [p_J(s_t), +\infty)$. Let $F(s_t, p)$ be any distribution function with support $\{p_1, ..., p_J\}$. Let $\Theta(s_t, p) = \infty$ for $p < p_1(s_t)$

and construct $\Theta(s_t, p)$ for $p \geq p_1(s_t)$ as the following

$$\Theta(s_t, p) = \sum_{j=1}^{J} \gamma_j(s_t, p) \cdot \theta_j(s_t) \cdot \frac{p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}{p - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}.$$

I check objects constructed above satisfy the equilibrium conditions in definition 1.

Equilibrium Beliefs: To simplify the illustration, let us consider the equilibrium beliefs condition first. Suppose $\Theta(s_t, p) < \infty$ and $\gamma_j(s_t, p) > 0$. By construction, I have $p \in [p_j(s_t), p_{j+1}(s_t))$ if $1 < j < J$; $p \in [0, p_2(s_t))$ if $j = 1$; and $p \in [p_J(s_t), +\infty)$ if $j = J$. For the case $1 < j < J$, it is sufficient to prove

$$\Theta(s_t, p)\Big(p - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big) \geq \Theta(s_t, p')\Big(p' - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big) \tag{52}$$

for all $p' \notin [p_j(s_t), p_{j+1}(s_t))$. Suppose $p' < p_j(s_t)$. There exists a $1 \leq j' < j$ such that $p' \in [p_{j'}(s_t), p_{j'+1}(s_t))$. It implies

$$\begin{aligned}\Theta(s_t, p')\Big(p' - \rho^l \cdot \mathbb{E}_t[V_{j'}^s(s_{t+1})]\Big) &= \theta_{j'}(s_t)\Big(p_{j'}(s_t) - \rho^l \cdot \mathbb{E}_t[V_{j'}^s(s_{t+1})]\Big) \\ &= \theta_{j'+1}(s_t)\Big(p_{j'+1}(s_t) - \rho^l \cdot \mathbb{E}_t[V_{j'}^s(s_{t+1})]\Big),\end{aligned}$$

where the first equality comes from the construction of $\Theta(s_t, p)$ function and the second equality uses the binding IC constraint between type j' and $j' + 1$. Thus, I obtain $\theta_{j'+1}(s_t) < \Theta(s_t, p') < \theta_{j'}(s_t)$. From Lemma 1, I get

$$[\theta_{j'+1}(s_t) - \Theta(s_t, p')] \cdot \rho^l \left[\mathbb{E}_t[V_{j'}^s(s_{t+1})] - \mathbb{E}_t[V_j^s(s_{t+1})]\right] > 0.$$

Subtracting this inequality from the above equation,

$$\begin{aligned}\Theta(s_t, p')\Big(p' - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big) &\leq \theta_{j'+1}(s_t)\Big(p_{j'+1}(s_t) - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big) \\ &\leq \theta_j(s_t)\Big(p_j(s_t) - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big) \\ &= \Theta(s_t, p)\Big(p - \rho^l \cdot \mathbb{E}_t[V_j^s(s_{t+1})]\Big).\end{aligned}$$

The second inequality is due to the fact that, as proved in Lemma 1, $\theta_{j'+1}(s_t)$ and $p_{j'+1}(s_t)$ are feasible for type j seller. The final equality is by definition of $\Theta(s_t, p)$. This verifies (52) if $p' < p_j(s_t)$.

The alternative case $p' > p_{j+1}(s_t)$ is similar. There exists a $j < j' \leq J$ such that $p' \in [p_{j'-1}(s_t), p_{j'}(s_t))$. Following the definition of $\Theta(s_t, p')$, I obtain

$$\Theta(s_t, p')\left(p' - \rho^l \cdot \mathbb{E}_t[V^s_{j'-1}(s_{t+1})]\right) = \theta_{j'-1}(s_t)\left(p_{j'-1}(s_t) - \rho^l \cdot \mathbb{E}_t[V^s_{j'-1}(s_{t+1})]\right).$$

This proves $\theta_{j'-1}(s_t) > \Theta(s_t, p')$, which implies

$$[\theta_{j'-1}(s_t) - \Theta(s_t, p')] \cdot \rho^l \left[\mathbb{E}_t[V^s_{j'-1}(s_{t+1})] - \mathbb{E}_t[V^s_j(s_{t+1})]\right] > 0.$$

Together with the IC constraint between type j and $j' - 1$, I have

$$\begin{aligned}
\Theta(s_t, p')\left(p' - \rho^l \cdot \mathbb{E}_t[V^s_j(s_{t+1})]\right) &\leq \theta_{j'-1}(s_t)\left(p_{j'-1}(s_t) - \rho^l \cdot \mathbb{E}_t[V^s_j(s_{t+1})]\right) \\
&\leq \theta_j(s_t)\left(p_j(s_t) - \rho^l \cdot \mathbb{E}_t[V^s_j(s_{t+1})]\right) \\
&= \Theta(s_t, p)\left(p - \rho^l \cdot \mathbb{E}_t[V^s_j(s_{t+1})]\right).
\end{aligned}$$

This proves what I want. For the case $j = 1$ and $j = J$, the proof is even simpler since I only need to consider one-sided case $p' > p_2(s_t)$ or $p' < p_J(s_t)$. They are exactly the same with the above. Notice that seller's optimality condition follows immediately after the verification of equilibrium beliefs condition.

Active Markets: Now I verify the active markets condition. It is sufficient to prove that $p_j(s_t)$ solves the maximization problem on the right hand side of (19) given functions $\lambda(s_t)$, $\Theta(s_t, p)$ and $\Gamma(s_t, p)$. Recall that from Lemma 4 and my construction, $\Theta(s_t, p) \leq 1$ for all p. Substituting $\Gamma(s_t, p)$ back, for all p such that $\gamma_j(s_t, p) > 0$ and $\Theta(s_t, p) < \infty$, I have $p \in [p_j(s_t), p_{j+1}(s_t))$. Given the construction $F(s_t, p)$, it is sufficient to consider the simplified version of *rhs* of (19):

$$\begin{aligned}
&\max_p \left[\sum_j \frac{\min\{\Theta^{-1}(s_t, p), 1\} \cdot \rho^h \mathbb{E}_t[v^b_j(s_{t+1})]}{p} \gamma_j(s_t, p)\right] \\
&= \max_j \left\{ \max_{p \in [p_j(s_t), p_{j+1}(s_t))} \frac{\rho^h \mathbb{E}_t[v^b_j(s_{t+1})]}{p} \right\} \\
&= \frac{\rho^h \mathbb{E}_t[v^b_j(s_{t+1})]}{p_j(s_t)} \\
&= \lambda(s_t),
\end{aligned}$$

for all $j \in \{1, 2, ..., J\}$. This confirms the active markets condition.

Other equilibrium condition are obviously satisfied. This completes the proof. □

Lemma 8. *For any state s_t, assume that $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$. Suppose the function $\lambda(s_t)$ is continuous in s_t and the transition function of aggregate state satisfies the Feller property. Let $\{V_j^s(s_t)\}_{j=1}^J$, $\Theta(s_t, p)$, $\Gamma(s_t)$, and $F(s_t, p)$ be a partial equilibrium on the asset market. Then there exists a $p_j(s_t)$ for all j and s_t such that $\gamma_j(s_t, p_j(s_t)) > 0$, if $\Theta(s_t, p_j(s_t)) > 0$, then $\{p_j(s_t), \Theta(s_t, p_j(s_t)), V_j^s(s_t)\}$ is a solution to problem $\mathcal{P}_j$.*

Proof. Fix any state s_t. From the consistency of supplies with beliefs condition, there exists a $p_j(s_t)$ such that $\gamma_j(s_t, p_j(s_t)) > 0$ for all j. Suppose that $\Theta(s_t, p_j(s_t)) > 0$. I denote this term as $\theta_j(s_t)$ hereafter. According to the equilibrium beliefs condition, $p_j(s_t)$ solves the maximization problem on the right hand side of (1) for seller j. As a preliminary, I first prove that $V_j^s(s_t) > V_{j'}^s(s_t)$ if $j > j'$. From the sequence form definition of $V_j^s(s_t)$, I obtain the following inequality. The first inequality holds since I have dropped the max operator and injected a particular sub-market $p_{j'}(s_t)$ in (1) for seller j. The strict inequality comes from the fact that $\delta_{t,j} > \delta_{t,j'}$ for all t and $j > j'$. The equality in the third line uses again the sequence form definition of $V_{j'}^s(s_t)$ and the seller's optimality condition.

$$
\begin{aligned}
& V_j^s(\delta_{t+l,1}, \lambda(s_{t+l})) \\
& \geq \mathbb{E}\Big[\sum_{l=0}^{\infty}\Big[\prod_{\tau=0}^{l}(1-\min\{\theta_{j-1}(s_{t+\tau}),1\})[\rho^l(1-\alpha)]^l \\
& \cdot \{\delta_{t+l,j} + \min\{\theta_{j-1}(s_{t+l}),1\}p_{j-1}(s_{t+l})\}\Big]\Big] \\
& > \mathbb{E}\Big[\sum_{l=0}^{\infty}\Big[\prod_{\tau=0}^{l}(1-\min\{\theta_{j-1}(s_{t+\tau})),1\})[\rho^l(1-\alpha)]^l \\
& \cdot \{\delta_{t+l,j-1} + \min\{\theta_{j-1}(s_{t+l}),1\}p_{j-1}(s_{t+l})\}\Big]\Big] \\
& = V_{j-1}^s(\delta_{t+l,1}, \lambda(s_{t+l})).
\end{aligned}
$$

With this preliminary result, I first prove that the policy $\theta_j(s_t), p_j(s_t)$ is feasible for problem $\mathcal{P}_j$.

I check the constraint (25) first. Suppose, for a contradiction, that (25) is violated for policy pair $\theta_j(s_t), p_j(s_t)$. Since sub-market $p_j(s_t)$ is active, from the buyer's optimality condition, there must be pooling in this sub-market to maintain (19). That is, there exists a $j' \neq j$ with $\gamma_{j'}(s_t, p_j(s_t)) > 0$ such that

$$\lambda(s_t) < \frac{\min\{\theta_j^{-1}(s_t), 1\} \cdot \rho^h \mathbb{E}_t[v_{j'}^b(s_{t+1})]}{p_j(s_t)} \\ + \rho^h(1 - \min\{\theta_j^{-1}(s_t), 1\}) \cdot \mathbb{E}_t[\lambda(s_{t+1})].$$

I must have $\mathbb{E}_t[v_{j'}^b(s_{t+1})] > \mathbb{E}_t[v_j^b(s_{t+1})]$. This implies $j' > j$. If $\theta_j(s_t) = \infty$, the above inequality degenerates to

$$\lambda(s_t) < \rho^h \cdot \mathbb{E}_t[\lambda(s_{t+1})],$$

which is a contradiction to $\lambda(s_t) > \underline{\lambda}(s_t)$. Hence $\theta_j(s_t) < \infty$. From the equilibrium beliefs condition, compared with any $p' \neq p_j(s_t)$, type j' seller prefers submarket $p_j(s_t)$, which suggests

$$\min\{\theta_j(s_t), 1\}(p_j(s_t) - \rho^l \mathbb{E}_t[V_{j'}^s(s_{t+1})]) \\ \geq \min\{\Theta(s_t, p'), 1\}(p' - \rho^l \mathbb{E}_t[V_{j'}^s(s_{t+1})]).$$

In particular, let us consider those submarket $p' > p_j(s_t)$. The above inequality implies $\min\{\Theta(s_t, p'), 1\} < \min\{\theta_j(s_t), 1\} < \infty$. I claim that if $\gamma_{j''}(s_t, p') > 0$, I have $j'' \geq j'$. To see this, consider the opposite case where $j'' < j'$. By the preliminary result I proved in the beginning of this Lemma, I obtain $V_{j''}^s(s_t) < V_{j'}^s(s_t)$. Therefore, from the single crossing condition, I get

$$\min\{\Theta(s_t, p'), 1\}(p' - \rho^l \mathbb{E}_t[V_{j''}^s(s_{t+1})]) \\ < \min\{\theta_j(s_t), 1\}(p_j(s_t) - \rho^l \mathbb{E}_t[V_{j''}^s(s_{t+1})]).$$

According to the equilibrium beliefs condition, $\gamma_{j''}(s_t, p') = 0$, which is a contradiction. Thus, by deviating to submarket p' that is slightly higher than $p_j(s_t)$, the buyer expects that he would encounter sellers with at least type j'. This delivers a strictly higher value than $\lambda(s_t)$, which is a contradiction.

Now I verify the constraint (26) is satisfied by the policy pair $\theta_j(s_t)$ with $p_j(s_t)$. This follows immediately from the seller's optimality condition. Thus, I have proved that $\theta_j(s_t), p_j(s_t)$ is feasible for problem $\mathcal{P}_j$.

Suppose there exists a state s_t and another policy pair $p'(s_t)$ with $\theta'(s_t)$ satisfying the constraint (25) and (26), and they achieve the optimality of $\mathcal{P}_j$ with a strictly higher payoff than $V_j^s(s_t)$. From Lemma 4, I obtain that $\theta'(s_t) \leq 1$. I derive a contradiction that buyers can deviate to a submarket $p''(s_t) < p'(s_t)$ and achieve a strictly higher value than $\lambda(s_t)$. First notice that for $p''(s_t)$ close enough to $p'(s_t)$, the policy pair $p''(s_t)$ with $\theta'(s_t)$ is feasible, and the value attained by this policy is strictly higher than $V_j^s(s_t)$. That is,

$$
\begin{aligned}
V_j^s(s_t) <& \Big\{\delta_{t,j} + \theta'(s_t)\Big[p''(s_t) - \rho^l \mathbb{E}_t\big[V_j^s(s_{t+1})\big]\Big] \\
&+ \rho^l(1-\theta'(s_t)) \cdot \mathbb{E}_t\big[V_j^s(s_{t+1})\big]\Big\}(1-\alpha),
\end{aligned}
$$

with

$$
\lambda(s_t) < \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{p''(s_t)},
$$

and for all $j' < j$

$$
\begin{aligned}
V_{j'}^s(s_t) >& \Big\{\delta_{t,j'} + \theta'(s_t)\Big[p''(s_t) - \rho^l \mathbb{E}_t\big[V_{j'}^s(s_{t+1})\big]\Big] \\
&+ \rho^l(1-\theta'(s_t))\mathbb{E}_t\big[V_{j'}^s(s_{t+1})\big]\Big\}(1-\alpha).
\end{aligned}
$$

From the seller's optimality condition, I obtain

$$
\begin{aligned}
V_j^s(s_t) \geq& \Big\{\delta_{t,j} + \Theta(s_t, p''(s_t))\Big[p''(s_t) - \rho^l \mathbb{E}_t\big[V_j^s(s_{t+1})\big]\Big] \\
&+ \rho^l(1-\Theta(s_t, p''(s_t))) \cdot \mathbb{E}_t\big[V_j^s(s_{t+1})\big]\Big\}(1-\alpha).
\end{aligned}
$$

Since $p''(s_t) < p'(s_t)$, I get $\Theta(s_t, p''(s_t)) < \theta'(s_t)$. Again, by the single crossing condition , all sellers with type $j' < j$ must strictly prefer $\theta'(s_t)$ with $p'(s_t)$ to the policy pair $\Theta(s_t, p''(s_t))$ with $p''(s_t)$. According to the equilibrium beliefs condition, $\gamma_{j'}(s_t, p''(s_t)) = 0$ for all $j' < j$. Since I have

$$\begin{aligned} &\frac{\rho^h \sum_{l=j}^J \gamma_l(s_t), p''(s_t)\mathbb{E}_t[v_l^b(s_{t+1})]}{p''(s_t)} \\ &\geq \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{p''(s_t)} \\ &> \lambda(s_t), \end{aligned}$$

buyers can achieve strictly higher value by deviating to the submarket $p''(s_t)$, which is a contradiction to the buyers' optimality condition. This completes the proof. □

Proof of Proposition 1

Proof. Suppose $\underline{\lambda}(s_t) < \lambda(s_t) < \bar{\lambda}(s_t)$ for all s_t. From Lemma 1 to Lemma 8, I have confirmed the existence and uniqueness of partial equilibrium on asset market and the characterization stated in proposition 1. Now I deal with the remaining cases where (i) $\lambda(s_t) = \bar{\lambda}(s_t)$ and (ii) $\lambda(s_t) = \underline{\lambda}(s_t)$ for some state s_t.

For the case (i), notice that type 1 seller is indifferent between selling the asset and keeping it to the next period. Let $p_j(s_t)$ for any j be the same with the construction in Lemma 7. I define $\Theta(s_t, p) = \infty$ if $p < p_1(s_t)$; $\Theta(s_t, p_1(s_t))$ be any number between $[0, 1]$; $\Theta(s_t, p) = \Theta(s_t, p_1(s_t)) \cdot \frac{p_1(s_t) - \rho^l \mathbb{E}_t[V_1^s(s_{t+1})]}{p - \rho^l \mathbb{E}_t[V_1^s(s_{t+1})]}$ if $p \in [p_1(s_t), p_2(s_t))$; and $\Theta(s_t, p) = 0$ for all $p \geq p_2(s_t)$. I remain the same construction for $\Gamma(s_t, p)$ as that in Lemma 7. Let $F(s_t, p_1(s_t)) = 1$ and $F(s_t, p) = 0$ for all $p \neq p_1(s_t)$. Given these constructions, following the exact same procedures as that in Lemma 6 and 7, I can define $V_j^s(s_t)$ for all j.

Now I verify these objects are indeed a partial equilibrium on the asset market. The equilibrium beliefs condition is immediate since for all types $j \geq 2$, the market tightness is 0 if $p \geq p_2(s_t)$, which implies they are indifferent between all these markets. For markets $p < p_2(s_t)$, the condition $\lambda(s_t) = \bar{\lambda}(s_t)$ suggests $p_1(s_t) = \rho^l \mathbb{E}_t[V_1^s(s_{t+1})] < \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]$. Hence it is a dominated policy for type $j \geq 2$ sellers to show up on submarket $p < p_2(s_t)$. The type 1 sellers are indifferent between all submarkets $p \geq p_1(s_t)$ and are strictly worse off in submarket $p < p_1(s_t)$. This proves the equilibrium beliefs condition. The seller's optimality follows immediately after the equilibrium beliefs condition. Notice that the only active market in this case is $p_1(s_t)$, and obviously $p_1(s_t)$ solves the maximization problem on the right hand side of (19). Other equilibrium conditions follow from the construction.

For the case (ii), I remain all the constructions in the Lemma 7 except that I modify $\Theta(s_t, p_1(s_t))$ to be an arbitrary number greater than or equal to 1. I can replicate the proof in Lemma 7 with no difficulty.

The final step is to prove that if $\Theta(s_t, p_1(s_t)) < 1$, which only happens under the condition $\lambda(s_t) = \bar{\lambda}(s_t)$, then $\Theta(s_t, p_1(s_t)) = 0$ for all $p \geq p_2(s_t)$. Suppose that, for a contradiction, $\Theta(s_t, p) > 0$ for some $p \geq p_2(s_t)$. Let us assume that $p \in [p_j(s_t), p_{j+1}(s_t))$ (by defining $p_{J+1}(s_t) = \infty$, this is without lose of generality). By my construction of $\Theta(s_t, p)$, I have $\Theta(s_t, p_j(s_t)) > 0$. Thus, from Lemma 8, I obtain that $\{p_j(s_t), \Theta(s_t, p_j(s_t)), V_j^s(s_t)\}$ is a solution to problem $\mathcal{P}_j$. Combining with the fact that $p_1(s_t) = \rho^l \mathbb{E}_t[V_1^s(s_{t+1})]$, constraint (26) between type 1 seller and type j seller implies

$$
\begin{aligned}
&V_1^s(s_t) = (1-\alpha)[\delta_{t,1} + p_1(s_t)] \\
&\geq (1-\alpha)[\delta_{t,1} + \min\{\Theta_j(s_t, p_j(s_t)), 1\} p_j(s_t)] + (1 - \min\{\Theta_j(s_t, p_j(s_t)), 1\}) p_1(s_t).
\end{aligned}
$$

It suggests $\rho^l \mathbb{E}_t[V_1^s(s_{t+1})] = p_1(s_t) \geq p_j(s_t) \geq \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]$, which is a contradiction to Lemma 1. This completes the proof. □

Proof of Proposition 2

Proof. Let $\left(\{V_j^s(s_t)\}_j, \Theta(s_t, p), \Gamma(s_t, p), F(s_t, p)\right)$ be a partial equilibrium associated with function $\lambda(s_t)$. Let $\left(\{\tilde{V}_j^s(s_t)\}_j, \tilde{\Theta}(s_t, p), \tilde{\Gamma}(s_t, p), \tilde{F}(s_t, p)\right)$ be another partial equilibrium associated with function $\tilde{\lambda}(s_t)$. Given any state s_t and j, assume that $\bar{\lambda}(s_t) > \tilde{\lambda}(s_t) > \lambda(s_t) > \underline{\lambda}(s_t)$ and $\bar{\lambda}(s_t') > \tilde{\lambda}(s_t') = \lambda(s_t') > \underline{\lambda}(s_t)$ for all $s_t' \neq s_t$.

I first prove that $\tilde{p}_j(s_t) < p_j(s_t)$ for all j. The result comes directly from the binding constraint (25) implied from Lemma 3 and the fact that $\theta_j(s_t) \leq 1$

according to the Lemma 4. I have

$$p_j(s_t) = \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{\lambda(s_t)}$$
$$> \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})]}{\tilde{\lambda}(s_t)} = \tilde{p}_j(s_t).$$

Now I prove that $\theta_j(s_t) \geq \tilde{\theta}_j(s_t)$ for all j. According to the result in Lemma 4, $\theta_1(s_t) = \tilde{\theta}_1(s_t) = 1$. I prove that, for any state s_t, the function $\frac{\theta_{j+1}(s_t)}{\theta_j(s_t)}$ is increasing in $\lambda(s_t)$. From the characterization in equation (49) and (50) proved in Lemma 7, I can write

$$\frac{\theta_{j+1}(s_t)}{\theta_j(s_t)} = \frac{p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}{p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}$$
$$= \frac{\rho^h \mathbb{E}_t[v_j^b(s_{t+1})] - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})] \cdot \lambda(s_t)}{\rho^h \mathbb{E}_t[v_{j+1}^b(s_{t+1})] - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})] \cdot \lambda(s_t)}.$$

It is immediate that $p_j(s_t)$ and $\theta_j(s_t)$ are differentiable with respect to $\lambda(s_t)$. Therefore, it is sufficient to prove that $\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)}[\frac{\theta_{j+1}(s_t)}{\theta_j(s_t)}] \geq 0$. By a simple calculation and rearrangement, I can write this derivative as

$$\frac{\dfrac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)}\left[p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]\right]}{p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]} - \dfrac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)}\left[p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]\right]}{p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}}{\left[p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]\right]^3 \left[p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]\right]}$$

Since the denominator of the above inequality is positive, it is sufficient to prove that

$$\frac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)} p_j(s_t)}{p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]} \leq \frac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)} p_{j+1}(s_t)}{p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]},$$

and

$$\frac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)} \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}{p_j(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]} \leq \frac{\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)} \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}{p_{j+1}(s_t) - \rho^l \mathbb{E}_t[V_j^s(s_{t+1})]}.$$

Notice that $\frac{\mathbf{d}}{\mathbf{d}\lambda(s_t)} \log p_j(s_t) = -1$ by the equation (49). Substituting this into the above inequalities and with a simple rearrangement, I obtain that both the two inequality are equivalent to $p_{j+1}(s_t) \geq p_j(s_t)$, which is true by the result in Lemma 4. This completes the proof. $\square$

C Partial Equilibrium on Repo Market

Proof of Proposition 3

Proof. Let's fix a repo contract. The proof is a replication of the argument in Morris and Shin (2001) proposition 3.1 (page 28). Let us consider the information extraction problem of lender i who receives $u_{i,t}$. Conditional on the information structure in the model, lender i's posterior distribution of μ_t is a normal distribution with mean $\frac{\sigma^2 \cdot 0 + \sigma_0^2 u_{i,t}}{\sigma^2 + \sigma_0^2}$ and variance $\frac{\sigma^2 \sigma_0^2}{\sigma^2 + \sigma_0^2}$. Let us denote this mean as $\tilde{\mu}_i$ and the variance as $\tilde{\sigma}_i^2$. For convenience, suppose that lender i believes all other lenders j follow a switching strategy at κ on their posterior belief $\tilde{\mu}_j$. That is, the lender j will accept the repo contract if $\tilde{\mu}_j > \kappa$, and choose the outside option otherwise. Thus, conditional on observing $u_{i,t}$, the expectation of the term $SC(f)$ is

$$\mathbb{E}\left[SC(f)|\kappa, u_{i,t}\right] = \int_{\mathbb{R}} SC\left(1 - \Phi\left(\frac{(1+\sigma^2/\sigma_0^2)\kappa - \mu}{\sigma}\right)\right) \mathbf{d}\Phi\left(\frac{\mu - \tilde{\mu}_i}{\sqrt{\tilde{\sigma}_i^2}}\right)$$

$$= \int_{\mathbb{R}} SC\left(1 - \Phi\left(\frac{(1+\sigma^2/\sigma_0^2)\kappa - x\cdot\sqrt{\tilde{\sigma}_i^2} - \tilde{\mu}_i}{\sigma}\right)\right) \cdot \phi(x)\mathbf{d}x,$$

where $\phi(x)$ is the density for the standard Normal distribution. Taking derivatives with respect to κ and $u_{i,t}$, I obtain

$$\int_{\mathbb{R}} SC'(f)\left[-\phi\left(\frac{(1+\sigma^2/\sigma_0^2)\kappa - x\cdot\sqrt{\tilde{\sigma}_i^2} - \tilde{\mu}_i}{\sigma}\right)\right] \cdot \left(\frac{1+\sigma^2/\sigma_0^2}{\sigma}\right)\phi(x)\mathbf{d}x$$

$$= \frac{\partial \mathbb{E}[SC(f)|\kappa, u_{i,t}]}{\partial \kappa} < 0 \tag{53}$$

and

$$\int_{\mathbb{R}} SC'(f)\left[\phi\left(\frac{(1+\sigma^2/\sigma_0^2)\kappa - x\cdot\sqrt{\tilde{\sigma}_i^2} - \tilde{\mu}_i}{\sigma}\right)\right] \cdot \frac{1}{\sigma}\left(\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\right)\phi(x)\mathbf{d}x$$

$$= \frac{\partial \mathbb{E}[SC(f)|\kappa, u_{i,t}]}{\partial u_{i,t}} > 0. \tag{54}$$

Notice that $\mathbb{E}(SC(f)|\kappa, u_{i,t})$ is continuously differentiable in κ and $u_{i,t}$, strictly decreasing in κ and strictly increasing in $u_{i,t}$. All other terms in V^l are constants conditional on the repo contract and s_t. It implies $V^l\left(s_t, R, (k_j)_j, \kappa, u_{i,t}\right)$ is strictly decreasing in κ and strictly increasing in $u_{i,t}$. Fix any state s_t. Let us denote $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$. I argue by induction that any strategy surviving n rounds of iterated deletion of (interim) strictly dominated strategies satisfies the following condition: choosing repo contract if the private utility is higher than $\bar{\xi}_n \cdot (1 + \sigma^2/\sigma_0^2)$ and choosing the outside option if $u_{i,t} < \underline{\xi}_n \cdot (1 + \sigma^2/\sigma_0^2)$. Obviously, the condition is true for any strategy if $n = 0$. Let us construct the sequence $\bar{\xi}_n$ and $\underline{\xi}_n$ recursively as follows

$$\underline{\xi}_{n+1} = \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2} \cdot \max\{x : V^l(s_t, R, (k_j)_j, \underline{\xi}_n, x) = 0\},$$

$$\bar{\xi}_{n+1} = \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2} \cdot \min\{x : V^l(s_t, R, (k_j)_j, \bar{\xi}_n, x) = 0\}.$$

It is obvious that $\underline{\xi}_n$ and $\bar{\xi}_n$ are well defined. I also observe that $\bar{\xi}_n$ is decreasing and $\underline{\xi}_n$ is increasing, which follows from the monotonicity of V^l in u_{it} and κ. Now, suppose that the condition is true for n. Again from the monotonicity of V^l in $u_{i,t}$, for an investor receiving $u_{i,t} < \underline{\xi}_{n+1} \cdot (1 + \sigma^2/\sigma_0^2)$, I get

$$V^l(s_t, R, (k_j)_j, \underline{\xi}_n, u_{i,t}) < 0.$$

This implies that choosing outside option is a dominated strategy for this investor. The argument is similar for the case that $u_{i,t} > \bar{\xi}_{n+1}$. Thus, there exists $\bar{\xi}$ and $\underline{\xi}$, defined as limits of $\bar{\xi}_n$ and $\underline{\xi}_n$ when n goes to infinity, such that

$$V^l(s_t, R, (k_j)_j, \bar{\xi}, \frac{\sigma_0^2 + \sigma^2}{\sigma_0^2} \cdot \bar{\xi}) = V^l(s_t, R, (k_j)_j, \underline{\xi}, \frac{\sigma_0^2 + \sigma^2}{\sigma_0^2} \cdot \underline{\xi}) = 0.$$

Next I prove that, under the assumption 1, $\bar{\xi} = \underline{\xi}$. To make the argument, it is sufficient to show that $\tilde{V}^l(y) \equiv V^l(s_t, R, (k_j)_j, y, \frac{\sigma_0^2+\sigma^2}{\sigma_0^2} \cdot y)$ is strictly monotone in y. Differentiating with respect to y and evaluating at $\kappa = u_{i,t} = y$:

$$\frac{\mathbf{d}\tilde{V}^l(y)}{\mathbf{d}y} = \int_{\mathbb{R}} \varphi \cdot \left[\frac{\partial SC(f)}{\partial \kappa} + \frac{\sigma^2 + \sigma_0^2}{\sigma_0^2} \cdot \frac{\partial SC(f)}{\partial u_{i,t}}\right] \mathbf{d}\Phi(x) + \frac{\sigma^2 + \sigma_0^2}{\sigma_0^2(1 - \rho^h(1-\beta))}$$

$$= \int_{\mathbb{R}} \left\{\varphi \cdot SC'(f) \cdot \phi(\cdot) \left(-\frac{\sigma}{\sigma_0^2}\right) + \frac{\sigma_0^2 + \sigma^2}{\sigma_0^2(1 - \rho^h(1-\beta))}\right\} \mathbf{d}\Phi(x)$$

Notice that $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$. The condition in assumption 1 is sufficient to guarantee that the term in the script bracket is strictly positive. This implies $\tilde{V}^l(y)$ is strictly increasing in y, hence $\bar{\xi} = \underline{\xi}$. Thus, the only symmetric strategy that survives the iterated elimination of dominated strategy is a switching strategy $\kappa^*(s_t, R^*, (k_j^*)_j)$ such that lenders will choose the repo contract if and only if they observe the random utility $u_{i,t}$ larger than $\kappa^* \cdot (1 + \frac{\sigma^2}{\sigma_0^2})$. The threshold κ^* is defined by the implicit function

$$\tilde{V}^l(\kappa^*) = 0.$$

This completes the proof. $\square$

D Details for Computation and Simulation

D.1 Equilibrium Computation

This section describes the solution method.[25] The model has several features that make the computation of the equilibrium is a challenging task. First of all, the dimension of the state space is high. As illustrated in previous sections, the aggregate state in my model is four-dimensional: $(B_t, K_{t,1}, K_{t,2}, \delta_{t,1})$. Secondly, three of my four-state variables are continuous. Thirdly, unlike the typical dynamic programming problem represented by the optimal growth model, my problem is equivalent to a stochastic dynamic game among three forward-looking players.

I employed the following strategy to address these issues. States are discretized with finite grid points. Values on the grid points are updated in the value function iteration. I use linear interpolations and Gauss–Hermite quadrature to calculate the off-grid expectations. And I utilize parallel computing whenever is possible to speed up the convergence. Roughly speaking, the algorithm is a modified Gaussian-Jacobi iteration. Unlike with a typical contraction mapping problem, the algorithm is not guaranteed to converge for an arbitrary initial guess. So the key to the success of computation is providing a good enough initial guess of the equilibrium. To get such a guess, I have created a chain of auxiliary models such that each one simplifies the previous model in a certain direction. The chain connects my benchmark model with a simple enough model that a solution is guaranteed. I solve the simplest model first; use its solution as the initial guess for the next model and iterates this procedure until the benchmark model is solved.

A similar technique is also used in the calibration. First, I start with a guess of parameters and a solved equilibrium associated with it. Then, I simulate the model (with details discussed in the next subsection) and get a simulated equilibrium path. Next, I update the guess of parameters by minimizing the distance between the moment targets and the simulated moments calculated from the aforementioned simulated path. Finally, I solve the equilibrium associated with the new parameters, using the equilibrium associated with the old parameters as the initial guess. It is obvious that if the iteration converges, I have obtained a local minimizer of the standard SMM objective function.

[25] The replication package, implemented with Julia, will be available soon on the author's website.

In this section, I take parameter values as given and elaborate the equilibrium solving algorithm. Notice that the state space X is continuous. The first step is to discretize the relevant region (to my simulations) of the state space. I consider $10 \times 100 \times 10 \times 10$ grids. The $\delta_{t,1}$ Markov process has a 10 points support. The state B_t is evenly gridded into 100 states between $0.15 \times \bar{B}$ and $0.6 \times \bar{B}$. For any j, the state for $K_{t,j}$ is evenly gridded with 10 points between $0.2 \times \frac{\bar{\theta}_j \cdot M_j}{\alpha}$ to $\frac{\bar{\theta}_j \cdot M_j}{\alpha}$. Equilibrium objects are updated on these grids in the iteration and off-grids values are calculated by linear interpolation.

Consider an initial guess of the equilibrium

$$\left(R^*(s_t), NR^*(s_t), \mathbb{E}_t[V_j^s(s_t)], \mathbb{E}_t[v_j^b(s_t)], \mathbb{E}_t[v_j^l(s_t)], \mathbb{E}_t[v_\pi^l(s_t)]\right).$$

It is updated by the following iteration. The convergence criterion is 10^{-4}.

if Convergence criterion passed **then**
    End the iteration
else
    for all s_t grids **do**
        Get $Cash(s_t)$ from $R^*(s_t)$ and $NR^*(s_t)$ by (11)
        Solve $\lambda(s_t)$, $p_j(s_t)$ and $\theta_j(s_t)$ from (49), (50) and (35)
        Get $c_j(s_t)$ by (2)
        Update $R^*(s_t)$ and $NR^*(s_t)$ by solving the problem in problem 1
        Get $T(s_{t+1}|s_t)$ from (18) and (22)
    end for
    Update value functions by (1), (5), (6) and (17)
    Calculate the distance between the updated equilibrium objects and the initial guess by sup norm
end if

The updating for value functions follow from the standard contraction mapping method.

D.2 Parameter Calibration

The detailed simulation method is reported in section 6.1. This section elaborates on how I implement the SMM for parameter calibration. Let $\mathfrak{P}$ denote the vector of parameters waiting for calibration.

$$\mathfrak{P} = \left\{ M_1, M_2, \rho^l, \nu_1, ..., \nu_4, \eta, \epsilon, \varphi, \sigma_0, \sigma \right\}.$$

Except for $\eta < 0$, all parameters are constrained to be non-negative. In addition, I require $\rho^l \in (0, \rho^h)$, and $(\varphi, \sigma_0, \sigma)$ jointly satisfy the assumption 1. Let us denote the feasible field of parameters as $\mathcal{C}$. Suppose the target moments described in section 5.3 is $Target_t$. Let $Sim_t(\mathfrak{P})$ be the simulated moments. The objective function is

$$\min_{\mathfrak{P}} \left[Sim_t(\mathfrak{P}) - Target_t\right]' \left[Sim_t(\mathfrak{P}) - Target_t\right] + Penalty \cdot \mathbb{1}_{\mathfrak{P} \notin \mathcal{C}}, \tag{55}$$

where $Penalty$ is a large enough constant. I use the Nelder-Mead method to solve the above optimization problem. Take a initial guess $\mathfrak{P}$ and its associated equilibrium $\mathcal{X}(s_t)$, the calibration algorithm is the following. The convergence criterion is 10^{-6}.

if Convergence criterion passed **then**
  End the iteration
else
  1. Given $\mathfrak{P}$, solve the equilibrium policy functions and value functions by method in appendix D.1 using $\mathcal{X}(s_t)$ as the initial guess
  2. Generate the simulated path for s_t
  3. Given s_t and value functions solved in 1, solve the optimization problem 55 and obtain optimal parameters $\mathfrak{P}'$
  4. Update $\mathcal{X}(s_t)$ by the equilibrium solved in 1
  5. Measure the distance between $\mathfrak{P}$ and $\mathfrak{P}'$
  6. Update $\mathfrak{P}$ by $\mathfrak{P}'$
end if

E Equilibrium with Cash Reserve

Proof of Proposition 4

Proof. The proof is by guess and verify. In the statement of the proposition, I have established a surjection $\mathcal{T}(\bar{s}_t)$ which defines any state $\bar{s}_t$ a corresponding state s_t in the state space of the benchmark model. There are three possibilities: (1) cash reserve is unused as buyers have sufficient fund to fulfill maturing repo liabilities; (2) cash reserve is sufficient to cover the shortfall and is partially used; (3) cash reserve is completely exhausted but still, there are some gaps. If case (1) occurs, the equilibrium outcome with the state $\bar{s}_t$ is observational equivalent to that when the state is s_t. If case (2) happens, the cash available (after applying cash reserves) for the buyer is simply 0. Case (3) is similar.

Given an equilibrium of the benchmark model, denoted by $\mathcal{X}(s_t)$[26], a candidate equilibrium $\mathcal{X}'(\bar{s}_t)$ for the model with cash reserve is well-defined by

$$\mathcal{X}'(\bar{s}_t) = \mathcal{X}(\mathcal{T}(\bar{s}_t)).$$

I verify the candidate equilibrium satisfies the functional equations determined by definition 3. Suppose the functional equation is denoted by

$$\mathfrak{F}(\mathcal{X}(s_t)) = 0, \quad \text{for all } s_t,$$

where $\mathfrak{F}$ contains equations (19), (1), (23), (5), (6), (17), (33), (29), (35), (18), and (22). Observe that $CR(s_t)$ only shows up indirectly in $\mathfrak{F}$ through the term *Cash*. By construction of the surjection $\mathcal{T}(\bar{s}_t)$, I obtain that the available cash for a buyer with state $\bar{s}_t$ is the same with that of buyer with state $\mathcal{T}(\bar{s}_t)$ in the benchmark model. Therefore, I have, for all $\bar{s}_t$,

$$\mathfrak{F}(\mathcal{X}'(\bar{s}_t)) = \mathfrak{F}(\mathcal{X}(\mathcal{T}(\bar{s}_t))) = 0.$$

This completes the proof. $\square$

[26] A tuple of functions $\left\{ \{V_j^s(s_t)\}_j, \Theta(s_t, p), \Gamma(s_t, p), F^*(s_t, p), \lambda(s_t), \kappa^*(\cdot), \left(R^*(s_t), (k_j^*(s_t))_j\right) \right\}$.